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The DEVELOPMENT of PRISMATIC GLASS BLOCK and the DAYLIGHTING LABORATORY

R. A. BOYD, Research Physicist

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The DEVELOPMENT *of* PRISMATIC GLASS BLOCK *and the* DAYLIGHTING LABORATORY

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The author, Dr. R. A. Boyd, is shown using an integrating-sphere type of reflectometer in the measurement of wall reflectivities.

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R. A. BOYD, Research Physicist

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Acknowledgments

The research and development work on prismatic glass block covered by this Bulletin and the establishment of the Daylighting Laboratory were made possible through the cooperation of the executive officers of the Owens-Illinois Glass Company and its subsidiary, the American Structural Products Company. The author wishes to express his appreciation to these companies and particularly to S. J. McGiveran, K. M. Henry, E. B. Dennis, Jr., Dr. H. B. Vincent, E. P. Lockart, W. E. McWhorter, R. C. Kendall and Murray McDavitt of the American Structural Products Company. The author is especially indebted to Dr. H. B. Vincent, now Director of Product Development for the American Structural Products Company, for his continued assistance and cooperation.

The research work was under the supervision of the Department of Physics of the University of Michigan until 1948. The author is grateful to Professor Emeritus H. M. Randall, Chairman of the Department until 1941, Professor E. F. Barker, present Chairman, Professor A. E. White, Director of Engineering Research Institute, and to Professor C. W. Good, Assistant Director of the Institute, for their interest and assistance in the project.

Photographs by Kenneth Hedrich, Hedrich Blessing Ltd., Chicago

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Introduction

The entrance of light-diffusing glass block into the field of fenestration* materials about fifteen years ago was a major factor in starting a series of improvements in the daylighting of buildings. One of the most significant developments in glass-block design, since the introduction of the original block, was the manufacture of the first prismatic glass block by the Owens-Illinois Glass Company in 1937. While these early prismatic glass block were more or less experimental, actual use indicated that they possessed interesting possibilities for daylight control, and considerable time and study was devoted to their improvement by the Company.

In order to secure outside assistance in the development of these light-controlling glass block, the Owens-Illinois Glass Company in 1940 started a program at the University of Michigan through the Engineering Research Institute for the purpose of establishing a daylighting laboratory and conducting a series of daylighting studies. In recent years this program has been continued by the American Structural Products Company, a subsidiary of the Owens-Illinois Glass Company.

The purpose of this Bulletin is to survey the research work that has been done on daylight control over the past ten years and to describe the laboratory and laboratory equipment that have been built for use in this work.

History and Development

The work was started with a study of the literature pertaining to natural illumination; this included the various lighting codes, the amount of light available in accordance with meteorological data, the methods employed for the daylighting of interiors, and the illumination-measuring instruments then in use. Some of this information is included in this section along with an outline of the initial activities of the project.

* Fenestration refers to any opening or arrangement of openings for the admission of daylight.

Lighting Codes

Although daylight, i.e., light from the sun and sky, has been admitted to building interiors through glazed and unglazed openings for centuries, it has only been in recent years that scientists, in general, have become sufficiently interested in the effects of illumination on human activity to study them and promote improved optical environments. This interest in the effects of light on human beings started about the time that the electric lamp came into general use.

In 1906 the Illuminating Engineering Society was organized for the advancement of the theory and practice of illuminating engineering and the dissemination of knowledge relating thereto. In its *Transactions* the Society has published, in addition to papers and discussions presented at its meetings, codes and standards for the lighting of various interiors. However, most of the early work done by the Society centered about artificial illumination. In fact, it was not until 1924 that the first I. E. S. committee on natural lighting was formed, and it was not until February, 1950, that the Society published the first *Recommended Practice of Daylighting* as prepared by this daylighting committee.

The first standards established for the daylighting of building interiors more or less followed those established for artificial illumination and called for a certain minimum number of foot-candles for various tasks, such as reading, without too much regard for quality. In an attempt to assure the proposed minimum intensities, window areas were specified as more or less fixed percentages of the floor area. Reduction in illumination due to different horizons, neighboring buildings, and the effect of different exposures was very seldom taken into account. Very little was said about the distribution of light throughout the interior, the elimination of high brightness ratios for the room occupant, and their dependence upon the reflectivities of the various room surfaces.

In recent years more and more consideration has been given to the quality of lighting, and today the characteristics of a good visual environment are stated in terms of both quality and quantity, with particular stress on the reduction of brightness ratios in the field of view and on the distribution of light throughout interiors. Recommendations covering these phases of building illumination have been included in the recently-published *Recommended Practice of Daylighting*[1], and, while an enumeration of all standards would be too space-consuming, some mention of a typical set of standards such as those set up for classroom illumination would be in order and would permit checking data presented on the following pages against these standards. The standards for classroom lighting recommend a minimum of 30 foot-candles for task illumination and brightness ratios, to be limited as follows:

1) Between the seeing task and immediately adjacent surfaces, such as between task and desk top, with the task the brighter surface—ratio of 1 to ⅓.

2) Between the task and the more remote darker surfaces in the surrounding field, such as between task and floor, and task and wall beneath the window—ratio of 1 to 1/10.

3) Between the task and the more remote brighter surrounding surfaces in the visual field, such as between task and ceiling—ratio of 1 to 10.

4) Between windows and surrounding surfaces adjacent to them in the visual field—ratio of 20 to 1.

The Daylighting Problem

As a preface to the presentation of the work done on this project, some discussion regarding the fundamental principles of daylight illumination of rooms will help to clarify the basic thinking in the development of controls for the daylighting of rooms.

Prior to the use of prismatic glass block as a fenestration material, the daylighting of building interiors was accomplished almost entirely through the use of nonfunctional light-transmitting materials, such as clear sheet glass, various diffusing and heat-absorbing glasses, and decorative glass block. These materials are nonfunctional in that they do not specifically control the direction or the amount of the incoming light.

In considering the daylighting of an interior, it is convenient to think of the total illumination of the task in two parts: the light arriving at the task directly from the fenestration and the light arriving after one or more reflections from the surrounding surfaces. This has an advantage from an analytical point of view in that the direct contribution can be calculated through the theory of surface sources (see Appendix A) and the diffuse component through the theory of inter-reflections. Obviously, it is the diffuse light scattered from the walls and ceiling that establishes surrounding brightnesses sufficiently high to bring the brightness ratios within acceptable limits.

Since the light from the sun and sky arrives at the building surfaces from above, except for an indefinite amount that is reflected from the earth and surroundings, the major portion of the light transmitted by nonfunctional materials, such as clear sheet glass, enters an interior in downward directions. The daylighting result in such a case is unsatisfactory in many respects.

The light is first incident on the working plane, the floor, or near-by surfaces, where it is absorbed to a large extent due to the lower reflectivities of these surfaces and the light-trapping ability of the furniture and occupants. This leads directly to an inefficient lighting system; large amounts of light may be admitted to an interior, but only small amounts are scattered about the room for diffuse lighting. In addition, the illumination of the working plane near the fenestration is high and decreases rapidly as the distance from the fenestration increases. This is true for the total illumination as well as the direct illumination. In the case of clear sheet glass the direct illumination is such a large percentage of the total that it is customary[1] when giving lighting predictions to cite only the direct components as given by the theory of surface sources (Appendix A). In fact, very little information can be found in the literature on the amount and distribution of the diffuse light in such a case.

The brightness ratios that the room occupant may encounter are very high. Between the sky as viewed through the clear glass and an adjacent wall the ratio may be as high as 2500[12] even if the room is decorated in light colors. Sufficient direct light from the sun or sky may fall directly on a room surface to constitute glare or establish a high brightness ratio.

Because of these acknowledged problems relating to nonfunctional fenestration materials considerable study in recent years has been given to the shielding of glass areas in an attempt to produce more uniform lighting and to decrease high brightness ratios. Shielding devices most commonly used are shades, Venetian blinds, and various louver arrangements. While some of these devices do protect the occupants of the room from high brightnesses, they create secondary problems, involving the adjustment of shades, lower light levels, and increased maintenance cost.

Design of Prismatic Glass Block

At the conclusion of the survey of various methods for illuminating rooms, it seemed apparent that the ceiling of a room was a surface very suitably located for the scattering of light to the working plane. Fig. A3 of Appendix A shows a projection of all surfaces of the room (shown by Fig. A4) in such a way that their relative lighting ability for a uniform brightness is indicated by the projected area. This diagram shows that from the standpoint of uniform brightness 62 per cent of the light reaching the working plane is contributed by the ceiling. Thus, it is apparent that a prismatic glass block designed to direct light to the ceiling for diffuse reflection would result in a lighting installation of high quality.

It was believed that, if a block could be designed to direct to the ceiling a large enough portion of the light which it transmitted, a low fenestration to task brightness ratio could be attained more efficiently than by any other known light-controlling device. After a preliminary study of the manufacturing process already established for the production of glass block it was thought that the very nature of the product offered an excellent opportunity for the successful control of light.

The very fact that the glass block was hollow furnished the designer with four surfaces on which light-controlling designs might be impressed in contrast to the two surfaces available in the case of sheet glass. Preliminary studies indicated that the basic principles incorporated in the original prismatic block designed by the Owens-Illinois Glass Company, i.e., the arrangement of horizontal prisms on the two inside surfaces and with vertical diffusing ribs on the exterior surfaces, were logical, and, consequently, the first design efforts were devoted to the refinement of prismatic design and to the improvement of the diffusing characteristics of the ribs on the exterior faces.

The first problem was to redesign the prisms on the interior faces of the block so that they would direct the major portion of the light above the horizontal, and, even though large amounts of light might be transmitted to the room, the block when viewed from below would be of a comfortable brightness. Because of the desirable high brightness of such a prismatic block in an upward direction, it became necessary to limit its use to that portion of the fenestration above eye level and to design another block of the light-diffusing type for use below eye level. As work continued, it became apparent that, while a solid glass-block panel made up of these two types of block was satisfactory for many types of buildings, certain other types, such as schools and offices, would be better served by substituting a strip of clear glass for the diffusing block to provide vision and ventilation. This combination of clear glass and light-directing glass block has become known to the public as the Insulux Fenestration System.

In contemplating the design of a prismatic glass block for best performance on sun exposures, the designer was faced with the extremely complex problem of handling wide variations in exterior illumination. For example, the exterior vertical surface illumination may range from 10,000 foot-candles on a clear day with direct sunlight to a few hundred foot-candles on an overcast day (see Appendix B). The designer was therefore faced with the problem of constructing a unit which would transmit an adequate amount of light from an overcast sky and yet not be excessively bright from observer positions when the sun was shining.

The brightness of an overcast sky is a maximum at the zenith, i.e., directly overhead, and decreases with altitude to about four-tenths of the zenith value at the horizon. Table D1 and Fig. B5 show this gradation of brightness as determined by measurement [2–3]. The variation of brightness and angle of incidence of the light from the sky makes the portion of the sky directly in front of the vertical surface at an altitude of about 30 degrees the greatest contributor to the vertical surface illumination (see Table D3). With this in mind, then, the prisms of the light-directing block were designed so that the block had its greatest percentage transmission for this direction. Fig. 1 shows vertical and horizontal sections of the prismatic block as finally developed and produced in commercial volume.

For overcast-sky conditions this block had a maximum brightness in a direction 20° above the horizontal; in fact, detailed measurements made in accordance with the methods outlined in Appendix D showed that 61 per cent of the total light transmitted emerged from the block above the horizontal.

Likewise, for direct sunlight the block always had its maximum brightness in a direction somewhat above the horizontal. A typical ray diagram showing the passage of light through the block prisms is presented in Fig. 2. For altitude angles at 0° azimuth with respect to the block, the directions of the maximum block brightness were as follows:

Sun Altitude	10°	20°	30°	40°	50°	60°	70°
Direction of Maximum Brightness	20°	24°	24°	27°	15°	9°	7°

An extensive series of measurements showed that for all possible exterior daylighting conditions this block always transmitted more than 60 per cent of the light into the region above the horizontal. It was found, too, that for all possible sun conditions the brightness of this block when viewed from below was within acceptable limits.

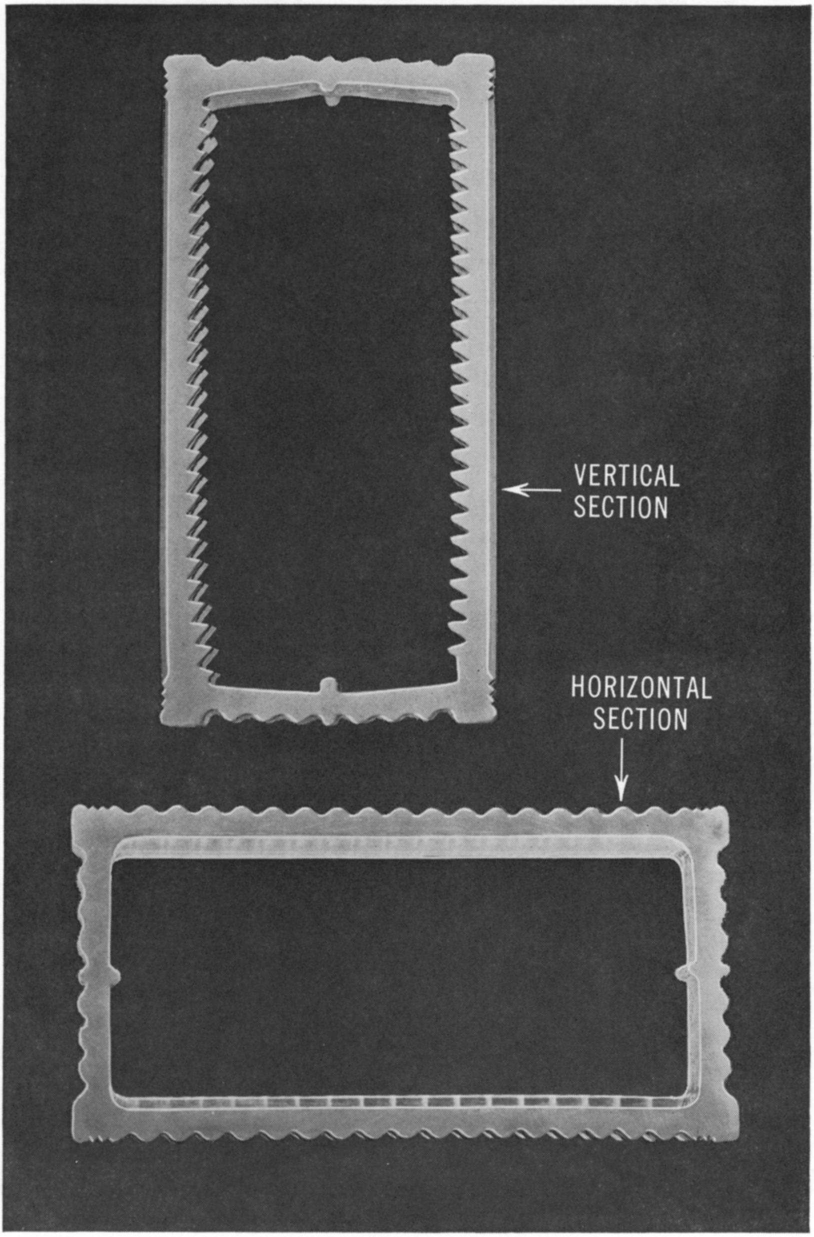


Figure 1. The ring-sections show the arrangement of the interior prisms and exterior face ribs of the first prismatic glass block that was developed through this research work.

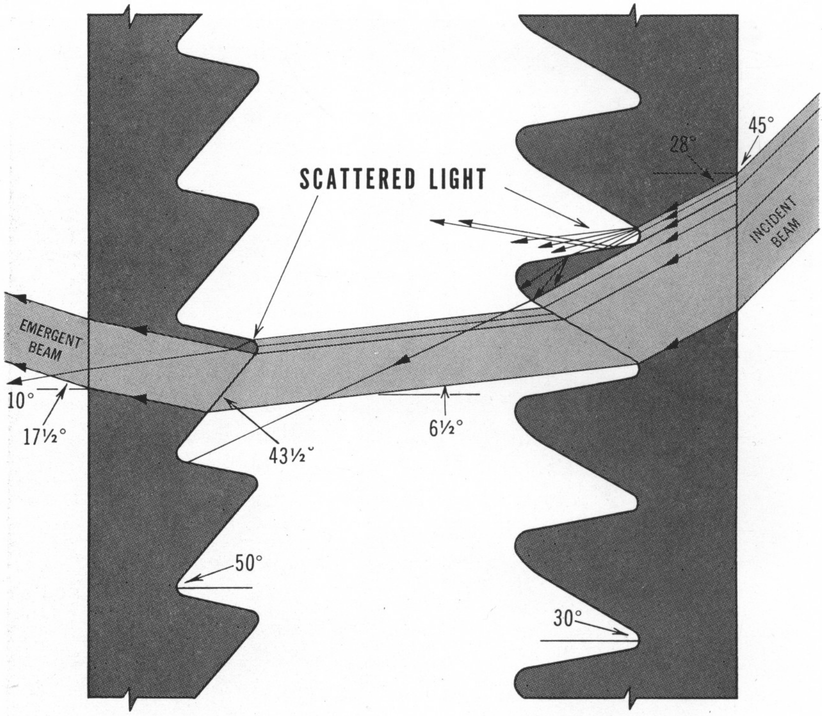


Figure 2. The major portion of a beam of light, such as direct sunlight, that is incident on the exterior face of a prismatic glass block at an angle of 45 degrees emerges from the interior face in a direction 17½ degrees above the horizontal; light emerging below the horizontal is due to scattering at radii and closure surfaces.

Measurements and Predictions

In order that one may predict the performance of a prismatic glass block or, in fact, any light-transmitting material under actual daylight conditions, measurements of brightness and transmission are required. The measurement of brightness must extend over all the directions from which an observer may view the material, and the distribution of light must be determined for the entire hemisphere of space into which the light is transmitted. These measurements must be made for a sufficiently large number of exterior daylighting conditions for the performance to be predicted for all possible installations.

Although prismatic glass block are designed for use with natural light, it was found impractical to use natural sources for the exact determination of block characteristics. The position of the sun, its brightness, and the brightness of the sky are continuously changing during any one day, thus making it impossible to obtain detailed, accurate measurements for a particular exterior daylighting condition. Also, it is necessary to wait at least several months at a particular latitude to obtain again the same conditions, and, of course, conditions for other latitudes and geographical locations cannot be obtained at all.

In view of these possible complications in using natural light for measurement purposes, it was decided at the start of this program that controllable artificial sources should be used to simulate the daylight sources. In doing this, the natural division was made by establishing an artificial sun and an artificial sky. Since meteorological data (Appendix B) and similar data were available to show the components of the total daylight illumination due to sky and sun, then the results obtained with the artificial sources could be adjusted in intensity and combined to satisfy any possible daylight condition. This plan proved to be quite satisfactory and is the one that is currently employed in the Laboratory.

The original laboratory for housing this equipment was located in the Randall Laboratory of Physics, and it was in this laboratory, using manually operated artificial lighting equipment, that the first measurements were made. These measurements were then used for the purpose of predicting building illumination and were checked by measuring installations in actual field jobs.

Even at this early date the system devised for predicting illumination was found to be acceptably accurate. The original laboratory, while crude in comparison to the one now in use, was valuable in that it served as a proving ground for the development of the product and for the development of the measuring methods now employed.

The Daylighting Laboratory

In view of the experience gained in the original laboratory it was decided, in the design of the new Laboratory, to continue to use artificial sources for the measurement of light transmission and brightness, and to incorporate a test room for the purpose of studying actual daylighting in order to eliminate the necessity of going into the field to check measurements and in order to make a more detailed study of actual job conditions. In selecting the location for the new Laboratory, it was believed that it should be located at an elevation high enough to eliminate usual ground obstructions and have a horizon of zero degrees.

After surveying the possible locations available at the University, it was decided to construct a "penthouse" on the roof of the East

Engineering Building. This building is high enough to obtain the desired zero horizon and, moreover, is fortunately oriented according to the points of the compass. Fig. 3 shows an exterior view of the completed Laboratory. The floor plan (Fig. 4) shows the location of the various rooms and the equipment which are described later in this Bulletin. The description of the equipment and its operation contained in this portion of the paper will be in the form of an outline only, and for information concerning its more technical and detailed aspects, reference may be made to the Appendices.

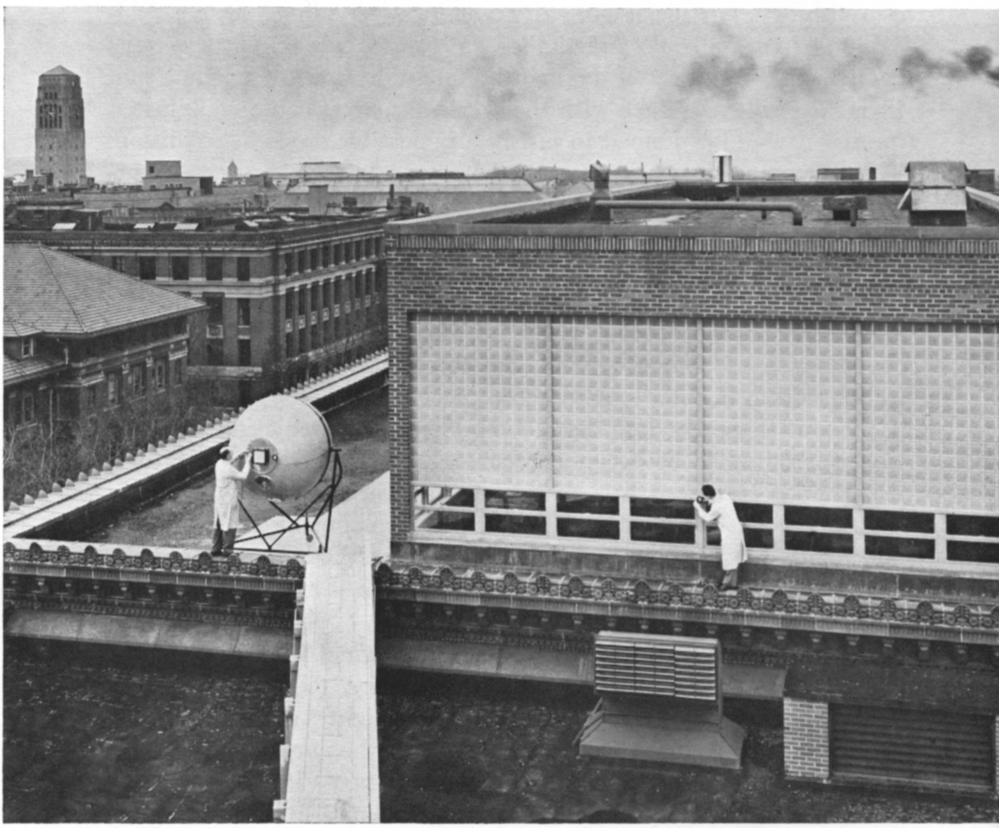


Figure 3. The Daylighting Laboratory at the University of Michigan is located on the roof of the East Engineering Building. The south and west exposure fenestrations, consisting of panels of a new type of prismatic glass block above clear sheet-glass vision strips, are used for the admission of daylight to a light-study room. A concrete platform on the west side allows the use of illumination measuring equipment with natural light.

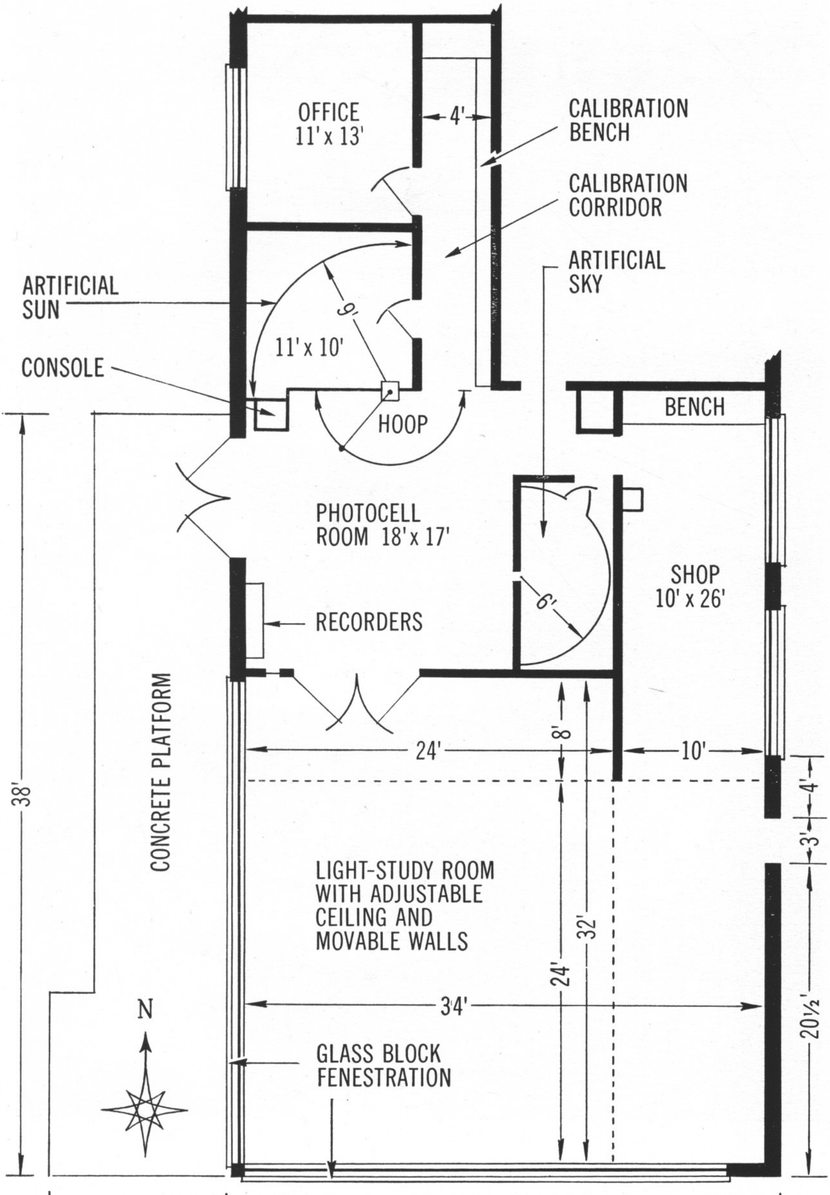


Figure 4. In addition to the experimental room for the daylighting studies, the Laboratory houses an artificial sun, an artificial sky, and photocell equipment for the measurement of block brightness and transmission. Auxiliary equipment is also available for calibration and other measurement purposes.

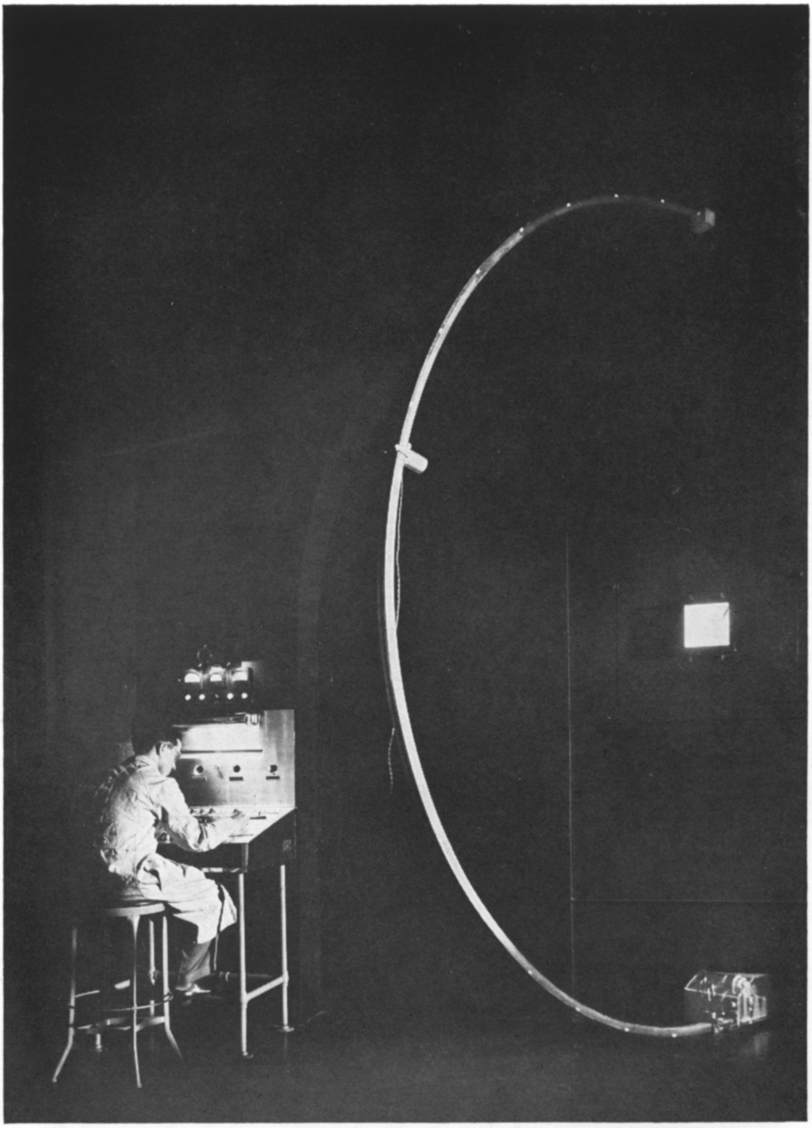


Figure 5. The brightness of a single glass block, or a section of other light-transmitting material, in any direction is determined by the reading of a photocell that faces the block from that direction. The photocell is moved along a semicircular hoop, by a cable drive, to different altitude positions, and the hoop is rotated about a vertical axis for different azimuth positions. The position of the photocell is controlled and determined from the control console.

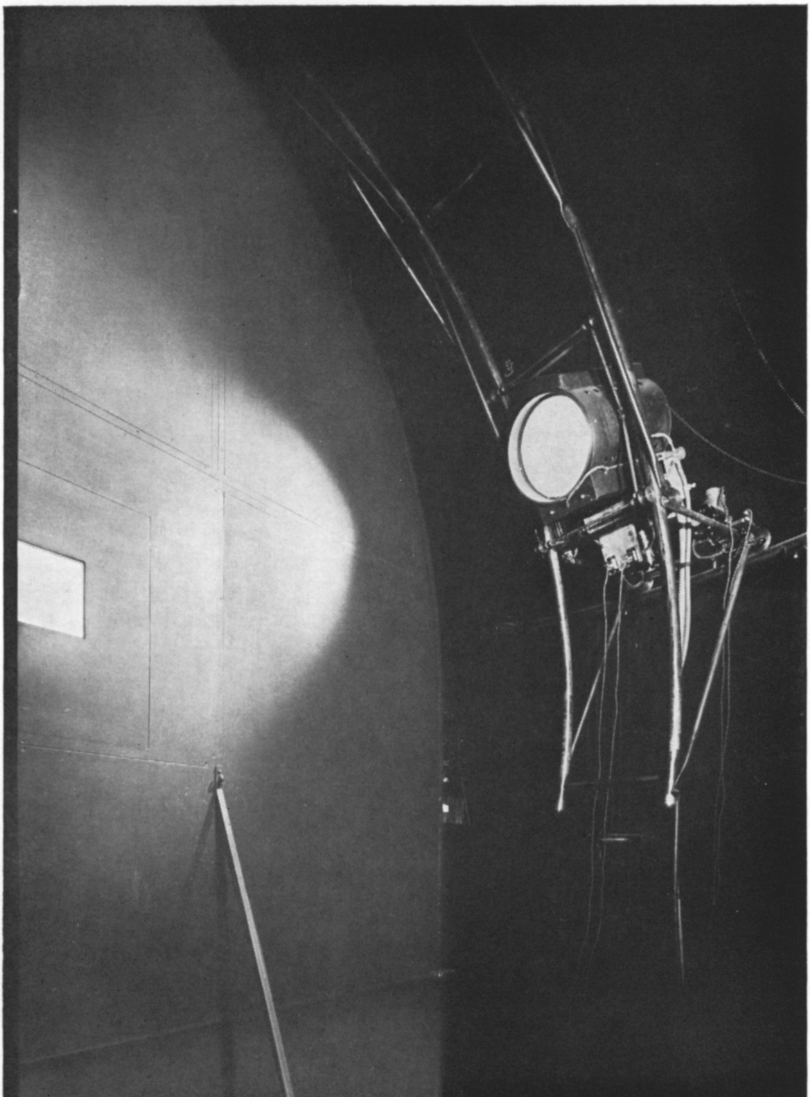


Figure 6. The exterior face of a glass block or other light-transmitting material is lighted by a beam of light from a 5000-watt projector or artificial sun. The angle of incidence of the beam of light is made to correspond to that of actual sunlight by moving the projector along vertical arcs to different altitude positions and by rotating the arcs about a vertical axis to different azimuth positions. The motor controls and the position indicators for the artificial sun are on the control console.

In the original laboratory the artificial sky, the artificial sun, and the photocell used for measurement of block brightness and the distribution of the transmitted light were incorporated in one unit. At that time it was thought necessary to establish a number of artificial skies in order to determine the block characteristics for the actual skies. Experience indicated that an artificial sky is needed only for demonstration and an occasional check on brightness and transmission, since the necessary block data for sky conditions can be obtained by calculation from the artificial sun data (see Appendix D). Because of this and other minor reasons the artificial sky in the present Laboratory is separate from the artificial sun and photocell arrangement.

Figs. 5 and 6 show the positions of the photocell and the artificial sun with respect to a glass block that is mounted in the wall dividing the two units. The metal plate on which the block is mounted can be exchanged for similar plates in order that block of other sizes and other fenestration materials can be investigated.

The artificial sun consists of a 5000-watt projector equipped with a 12-inch Fresnel lens. The lamp of this projector is movable with respect to the lens in order that the emergent light can be confined to a collimated beam or a somewhat divergent beam as indicated by Fig. 6. This projector is geared to a double arc in a vertical plane for setting at different altitude angles, and the entire unit is capable of rotating about a vertical axis for different azimuth angles, the center of curvature of these motions being at the center of the block face. The room that houses this unit is painted flat black in order to minimize the amount of scattered light.

The hoop on which the photocell is movable for different altitude positions is capable of rotating about a vertical axis, passing through the center of the inside face of the block and the center of curvature of the hoop, for different azimuth positions. The photocell is mounted on the hoop in such a manner that it directly faces the center of the inside face of the glass block.

The two motions of the artificial sun and the two motions of the photocell are independently controlled by four d-c motors. The switches for these motors and the rheostats for speed adjustment are mounted on the control console shown by Figs. 5 and 7. Four Selsyns geared to these four d-c motors with companion Selsyns mounted on the back of the control console indicate the positions of the artificial sun and the photocell. With these controls and indicators an operator can set the artificial sun and the photocell at any desired altitude and azimuth positions without leaving the control console.

Figure 7. In addition to the motor controls and position indicators for the photocell and the artificial sun the control console incorporates a resistance bridge from which the brightness values are determined.

With this equipment a glass block or a section of any light-transmitting material can be illuminated by a beam of light from a particular direction and the brightness of the inside face determined for any direction in the hemisphere of space into which the light is transmitted. For details of the method of measurement reference may be made to Appendix D.

The brightness of the block in a particular direction is a measure of the amount of light that is transmitted into a small cone of space centered about that direction. If a large number of such measurements are made, then the distribution of light transmitted by the block is determined. Such a distribution is needed in order to determine, by calculation, the illumination and brightness in a room that is fenestrated with the block.

If one is interested only in the total amount of light transmitted by the block, then the large integrating sphere is used. This sphere, shown by Fig. 8, indicates the total amount of light admitted to its interior through a glass block or other light-transmitting materials by a single reading from a photocell that faces the interior walls of the sphere. The operation and theory of the integrating sphere are given in Appendix C. This sphere can be used in conjunction with the artificial sources or with daylight. Fig. 9 shows a glass block being mounted for lighting by a laboratory light source prior to a measurement of total transmission with the integrating sphere. Fig. 10 shows the sphere being used outdoors to measure the total amount of daylight transmitted by a glass block. By using other mounting plates on the sphere, measurements of total transmission can be made on other-sized glass block and sections of other light-transmitting materials. When the sphere is used with the artificial sources, the readings are made on the control-table bridge by the methods outlined in Appendix D. When natural light is used, however, the sphere photocell and the photocell used for measurement of the exterior illumination are generally connected to the Speedomax Recorder and the readings recorded automatically.

The portion of the Laboratory occupied by the artificial sky is shown on the floor plan (Fig. 4). An overcast sky is simulated by this source in that the brightness of the inside surfaces varies about as indicated by Table D1. The horizon of the artificial sky is horizontally in line with the center of the block, and the region below this level has a reflectivity corresponding to that of the average earth surroundings. The interior of the artificial sky is lighted through the use of one 5000-watt incandescent lamp, and, when the lamp is new, the illumination of the exterior face of the block is about 500 footcandles.

The calibration of illumination instruments is a very essential part of the work of the Laboratory. In order to make sure that all measurements are made accurately, the calibration of the integrating sphere and of numerous photocells and brightness meters must be checked periodically. The voltage supplies, meters, and calibration

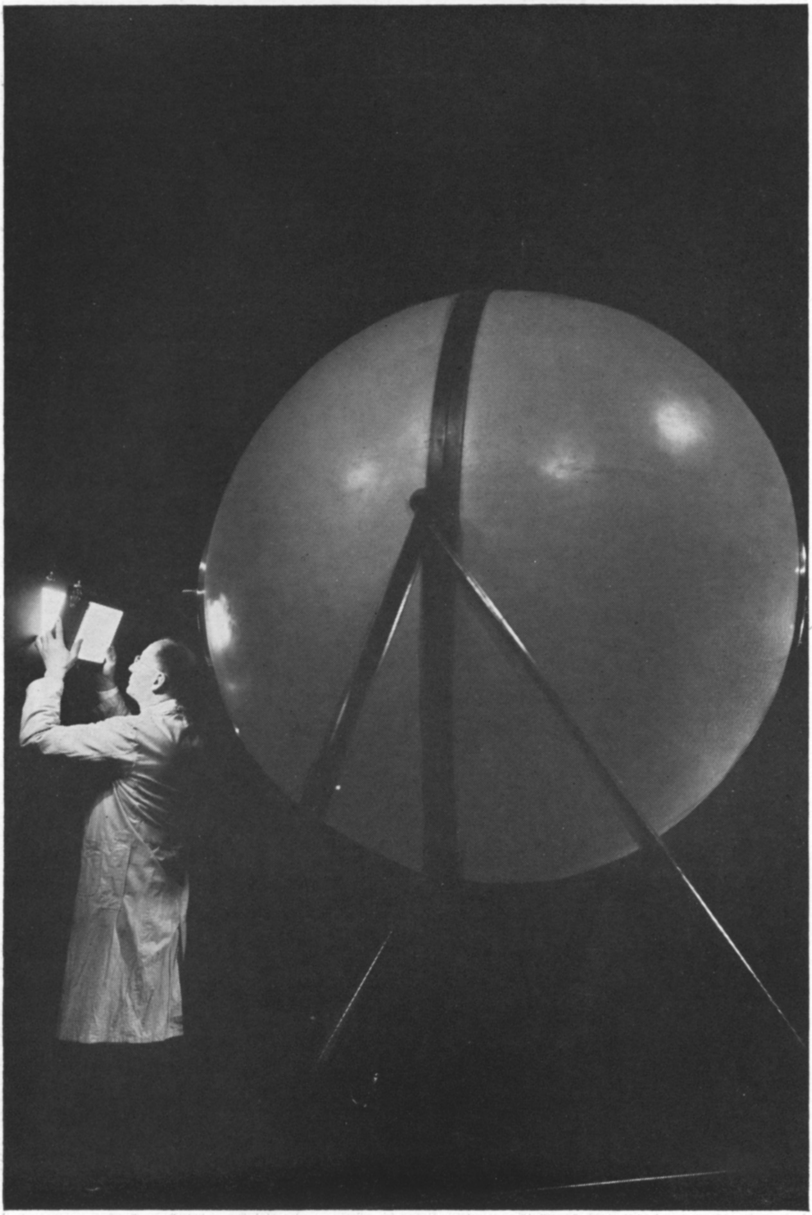


Figure 8. The integrating sphere is used for the measurement of the total amount of light that is transmitted by a single glass block or a section of other light-transmitting material from the artificial sun or artificial sky.

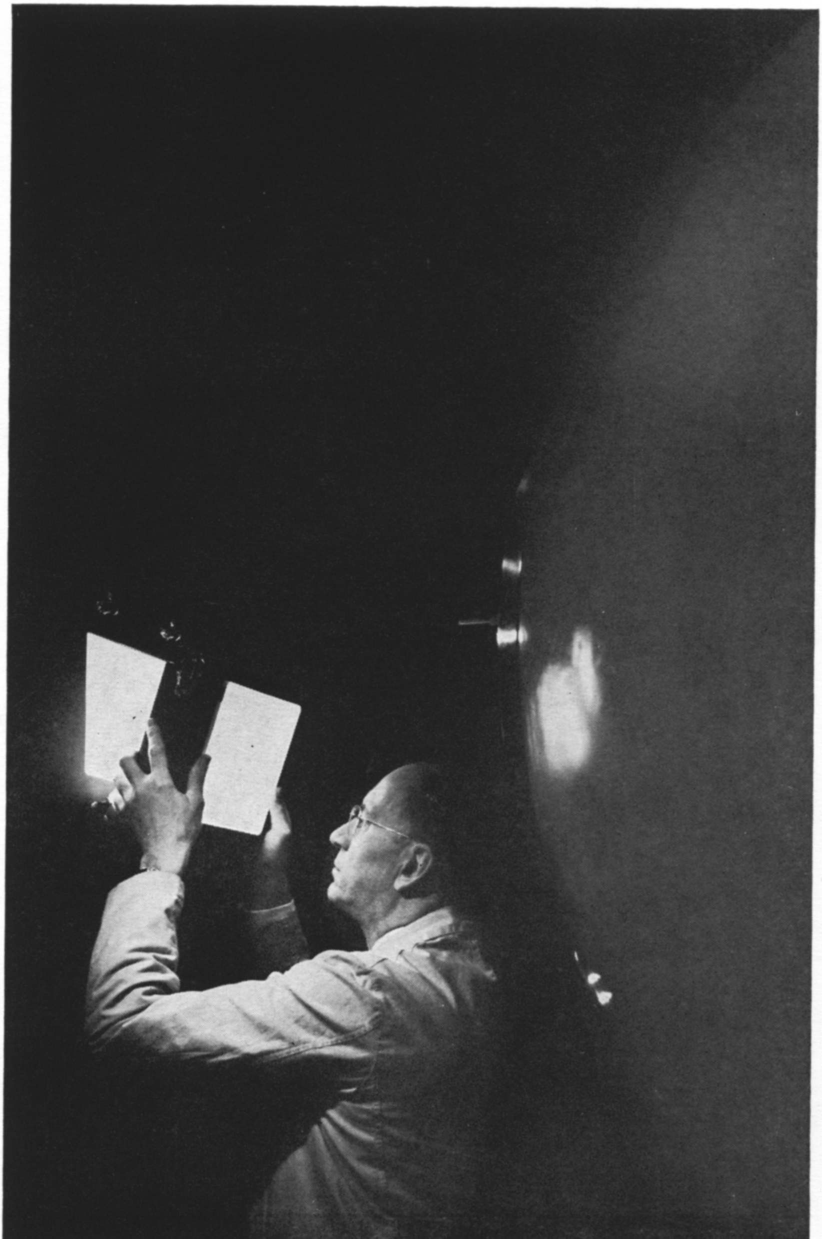


Figure 9. For measurement purposes, a single glass block is clamped in a frame before being mounted in the wall for illumination by the artificial sun or artificial sky.

Figure 10. The integrating sphere can be used outdoors with natural light. The total amount of light transmitted by the material is measured by a photocell that faces the interior walls of the sphere, and the exterior illumination of the material is measured by a photocell that faces the same direction as the material itself. The two photocell responses can be measured on the resistance bridge of the control console or they can be recorded automatically by the Speedomax Recorder.

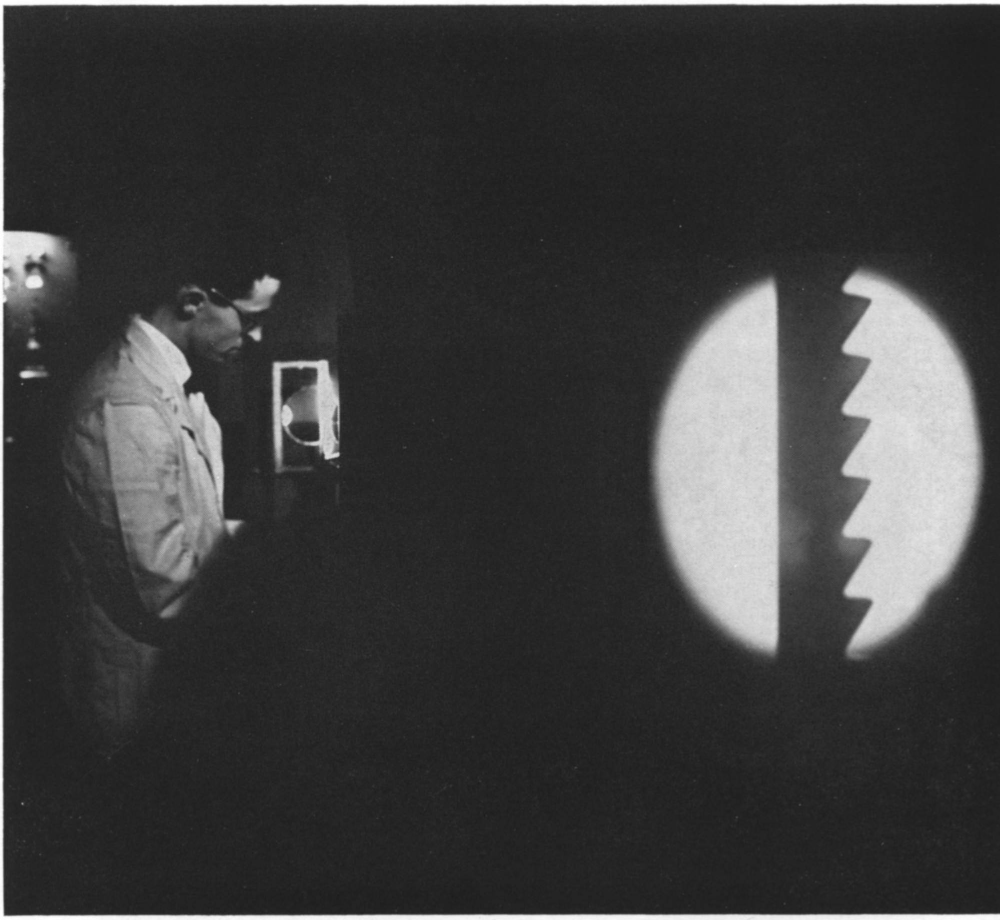


Figure 11. With the aid of the projection equipment in the calibration corridor, enlarged images of glass block prisms can be formed on an opal glass screen for the measurement of prism size and contour.

lamps required for this work are available at the north end of the calibration corridor (see floor plan). An optical bench along the east side of the corridor allows accurate measurement of distance from a standard lamp for establishment of definite illumination intensities. This equipment is used also for the investigation of prism cross-sections and the tracing of rays of light through prism sections. Fig. 11 shows how an enlarged image of a prism section of a glass block is projected onto an opal glass screen for measurement of size and prism angles.

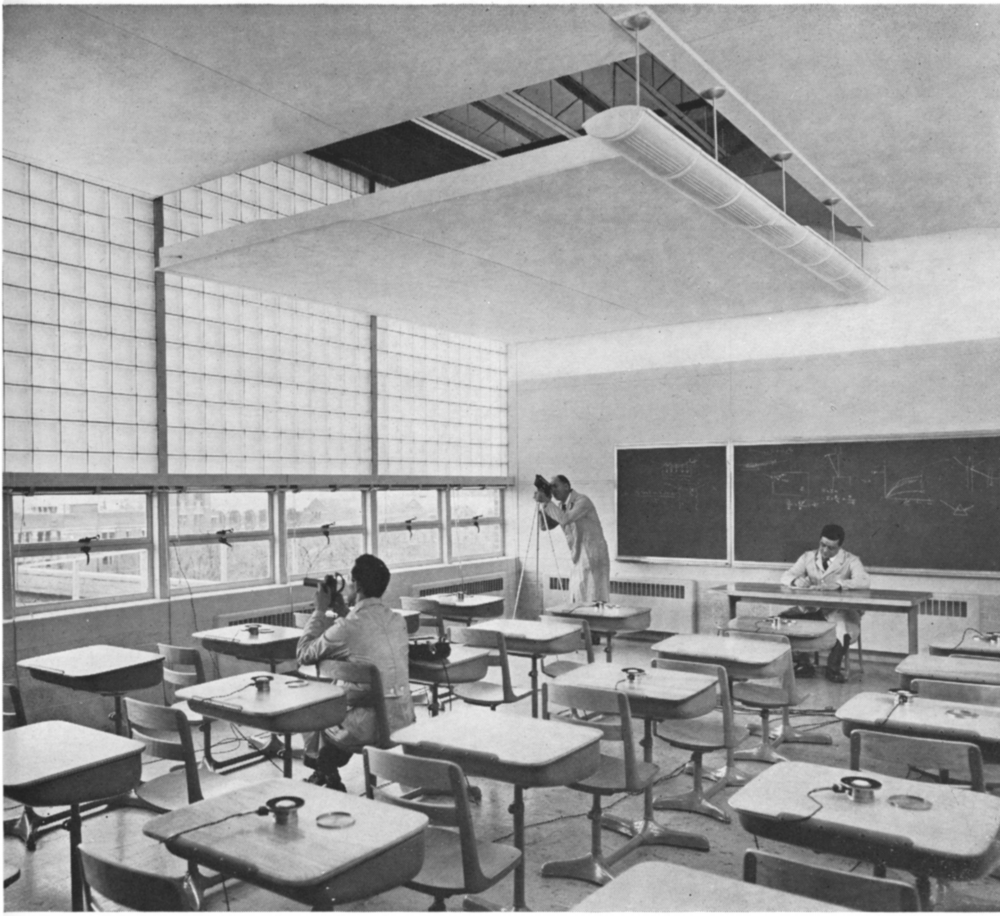


Figure 12. A full-size classroom with a single fenestration on south exposure can be arranged in the light-study room for detailed measurement of the quality and quantity of the daylighting.

The room that was included in the Laboratory for the study of daylighting, as can be seen from the floor plan (Fig. 4) is 32 feet by 34 feet with the exception of the northeast corner, 10 feet by 8 feet, which is a part of the workshop. This room has two fenestrations, one with south exposure and the other with west exposure, each fenestration consisting of a prismatic glass-block panel, 14 courses high and 48 block wide, above a clear, sheet-glass vision strip.

An experimental classroom has been constructed within this room

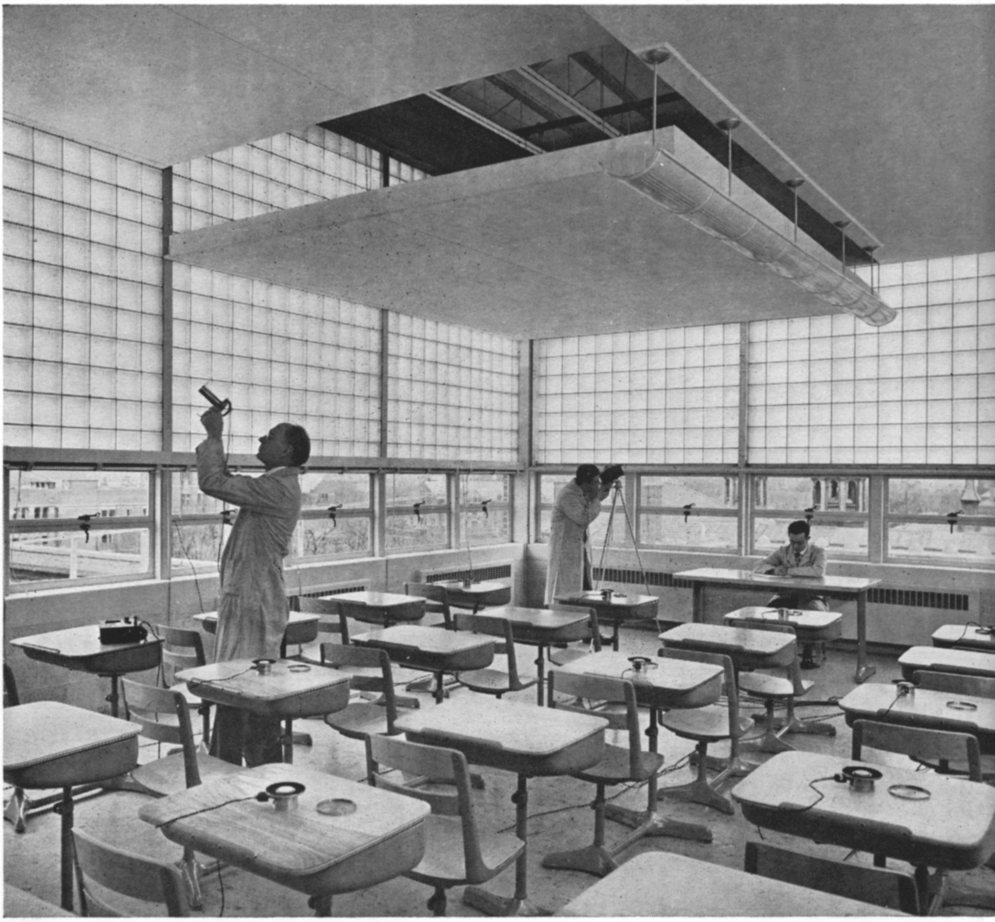


Figure 13. The classroom can be provided with bilateral daylighting by removing the cover panel from the west fenestration. The ceiling, consisting of four sections, is adjustable in height from 10 feet to 15 feet.

by suspending interior wall sections and ceiling panels from the roof trusses of the building. Each wall panel is 4 feet wide and 15 feet high, and each ceiling section is 16 feet by 12 feet. The classroom shown by Fig. 12 is 32 feet by 24 feet and is unilaterally lighted by the south fenestration. The west fenestration has been provided with a cover to form the west wall of the room. The ceiling sections are suspended by steel cables, which in turn are attached to winches, so that the ceiling can be adjusted in height from 10 feet to 15 feet. Since each section is suspended by four cables, the room can also be

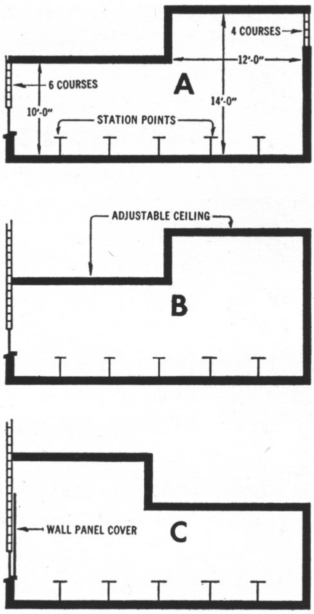


Figure 14. Section "A" shows classroom submitted for evaluation. The room features primary fenestration of vision strip and light-directing glass block in the outside (left) wall and as secondary fenestration a clerestory of light-directing glass block in the opposite wall. Light contribution of each fenestration is to be measured separately. In Section "B" the room is constructed with clerestory eliminated. Mounted in the outside wall is a vision strip surmounted by a glass-block panel. Ceilings are adjustable canvas flats while other walls are movable panels. Readings are taken throughout the room to measure light contribution from this fenestration. Section "C" shows room reversed to measure clerestory light contribution. This section is constructed against the outside wall with all fenestration save clerestory shielded out. Readings are taken at points in "C" identical with respective points in "B". Measurements are combined to obtain the total light contribution.

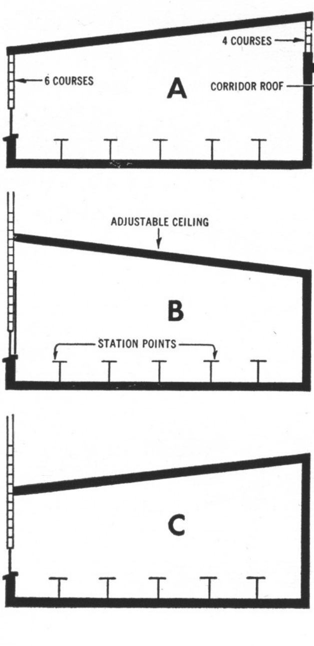


Figure 15. Again, Section "A" is the classroom to be evaluated. Primary fenestration is obtained from vision strip and glass block panel in the outside (left) wall and secondary fenestration from a clerestory in the opposite wall. Section "B" shows room direction and roof pitch reversed, with the outside wall appropriately shielded to reproduce the clerestory. Light contribution from this source is measured at points throughout the room. In Section "C" the room resumes its original direction, shielding is removed to simulate the primary fenestration, and contribution of light from this source is measured. Readings are combined to obtain the total contribution of light from all fenestration. The effects of reflected light from nearby roof surfaces are included by the use of auxiliary full-sized shields, connected to the weather side of the fenestration wall. Corridor roofs, and walls and roofs of opposite classrooms are typical examples of this reflected light.

provided with a sloping ceiling. This adjustable ceiling has been provided in order that the dependence of daylighting on ceiling height and contour can be studied.

As Fig. 13 indicates, the west fenestration cover can be easily removed in order to provide a fenestration on two adjacent walls. By transferring the west panel cover to the south fenestration and by shifting the ceiling and wall sections, the same classroom arrangement can be provided with a west-exposure fenestration.

Since the west fenestration with direct sunlight can be considered as equivalent to an east fenestration with direct sunlight, and since the west fenestration during the morning hours can be considered to function as a similar fenestration on north exposure, it is possible to simulate the daylighting of rooms oriented to all four points of the compass. For example, if one wishes to study the daylighting of a room that has a main fenestration on the south and a clerestory on north exposure, two sets of measurements would be made and the results totaled. The first set of measurements would be made on the south exposure with a portion of ceiling at the low level and the balance at the higher level. The second set would be made using the west fenestration during morning hours to represent a north exposure, the clerestory being simulated by obscuring the south panel and the lower west panel and by readjusting the ceiling. When measurements are completed, components from both sets are totaled to obtain task illumination and brightness produced by the combination fenestration. (See Figs. 14 and 15).

Although the daylighting of various full-scale rooms having different fenestration arrangements can be measured and studied in this manner, it is often expedient to make preliminary investigations through the use of models. A part of the first studies of classroom daylighting were made in a quarter-scale model. Through the use of this model the influence of different fenestration arrangements and surface reflectivities on the daylighting were easily studied. It is anticipated that continued use will be made of models, since in the present Laboratory they can be lighted with either artificial or natural sources.

Measurement Procedures

All illumination measurements are made with a photocell, shown by Fig. 16, which obeys the cosine law of illumination, i.e., it measures illumination accurately whether it is due to direct sunlight, light from a sky, or diffuse light in a room interior. An ordinary uncorrected barrier-layer photocell in measuring illumination due to direct

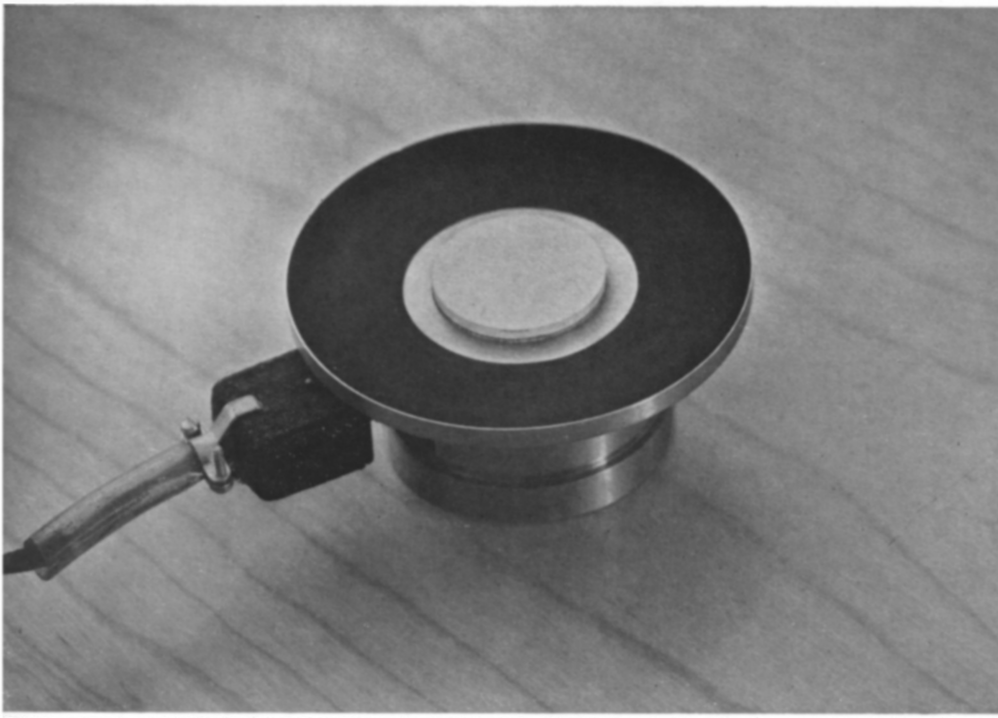


Figure 16. All measurements of illumination are made with a photocell, developed in the Laboratory, that obeys the cosine law of illumination The errors in measurement for the majority of illumination conditions encountered are not greater than 2 per cent.

sunlight may be in error by as much as 100 per cent, and in measuring diffuse light, it may be in error by as much as 25 per cent. The corrected photocell that is used in this work is accurate to within 1 per cent unless all the light is incident from an angle of about 80 degrees, when the error could be 4 per cent; for most exterior measurements involving direct sunlight and light from a sky, the error is not greater than 2 per cent. The construction and performance of this corrected photocell is discussed in Appendix C. This photocell was developed during the first few years of the project and has been used extensively for laboratory and field measurements.

For the measurement of desk-top illumination in the experimental classroom, numerous photocells are distributed throughout the room, as can be seen from Fig. 17. These photocells, as well as the outdoor photocells used for the measurement of the exterior vertical surface illuminations, are connected through a series of shunts to a

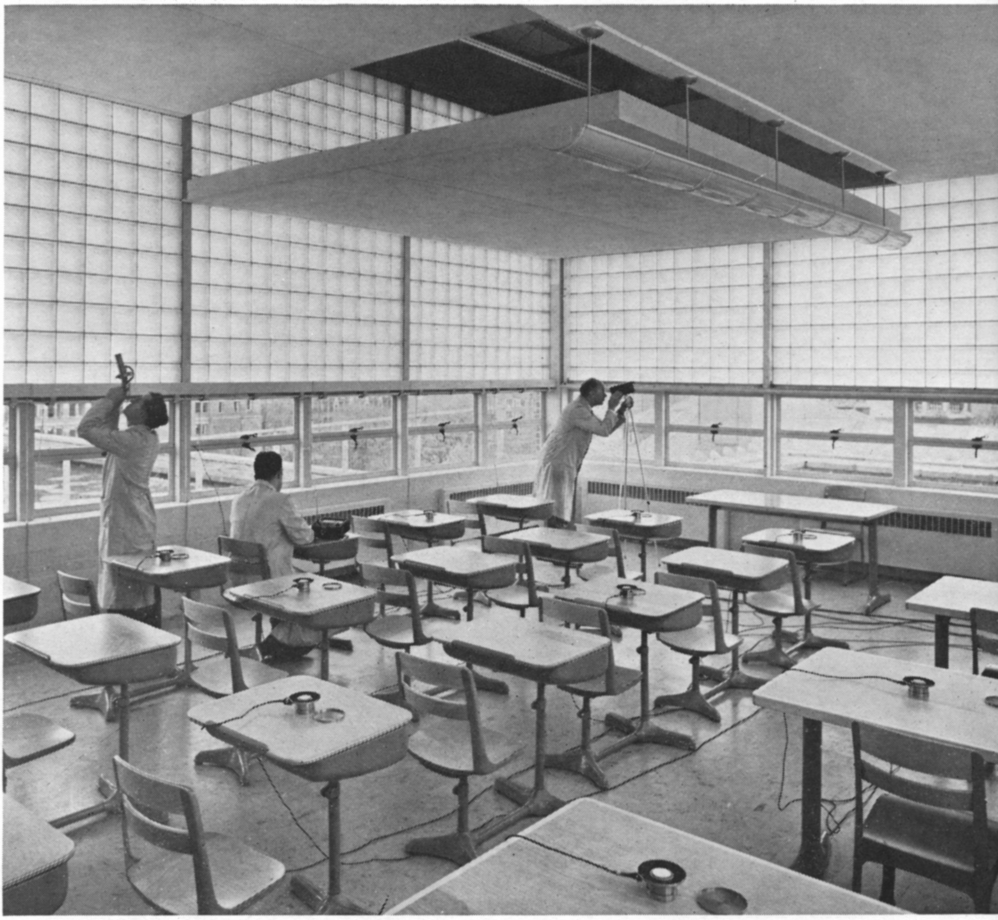


Figure 17. Illumination measurements in the light-study room are recorded automatically by connecting numerous photocells to the Speedomax Recorder. Brightnesses of the glass block panel, the vision strip, and all room surfaces are measured with a Luckiesh-Taylor Brightness Meter or a direct-reading brightness meter.

Speedomax Recorder. The Recorder and the panel of shunts can be seen in Fig. 18 and this Recorder is constructed so that responses of 16 photocells can be recorded automatically. Its speed is such that 4 seconds is required to record a single illumination figure. Thus, a second reading for the same photocell is recorded 64 seconds later. Each circuit has five shunts, thus providing each photocell with a wide range of measurement.

Figure 18. Illumination and total transmission results are recorded automatically by a Speedomax Recorder that has 16 independent circuits; four seconds are required for the recording of each figure.

In measuring the daylighting of a building interior, it is necessary to measure the exterior illumination at the same time in order to evaluate properly the results. Also, unless the sky is cloudless or densely overcast, the exterior illumination is likely to change appreciably during one set of measurements due to shifting clouds. A measure of the exterior illumination then allows one to adjust the interior illumination to a common exterior figure. Frequently one finds in

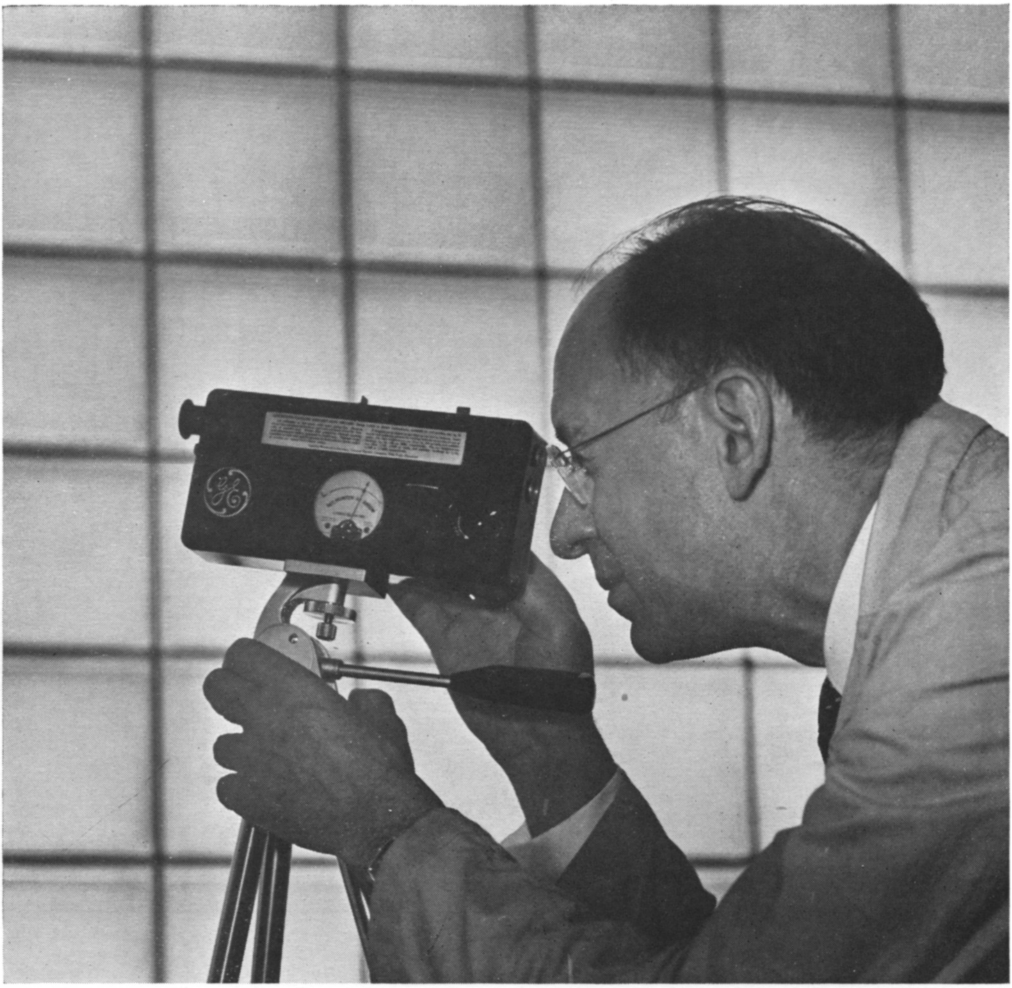


Figure 19. The Luckiesh-Taylor Brightness Meter is normally used for the measurement of the brightness of room surfaces.

published illumination data that the exterior illumination has been checked by the measurement of sky brightness or by the measurement of the illumination of a horizontal surface at the sill or just outside the fenestration.

The use of sky brightness as a control when the fenestration consists of clear, sheet glass is not too inaccurate since the daylighting of the room is primarily due to the brightness of the sky that is visible

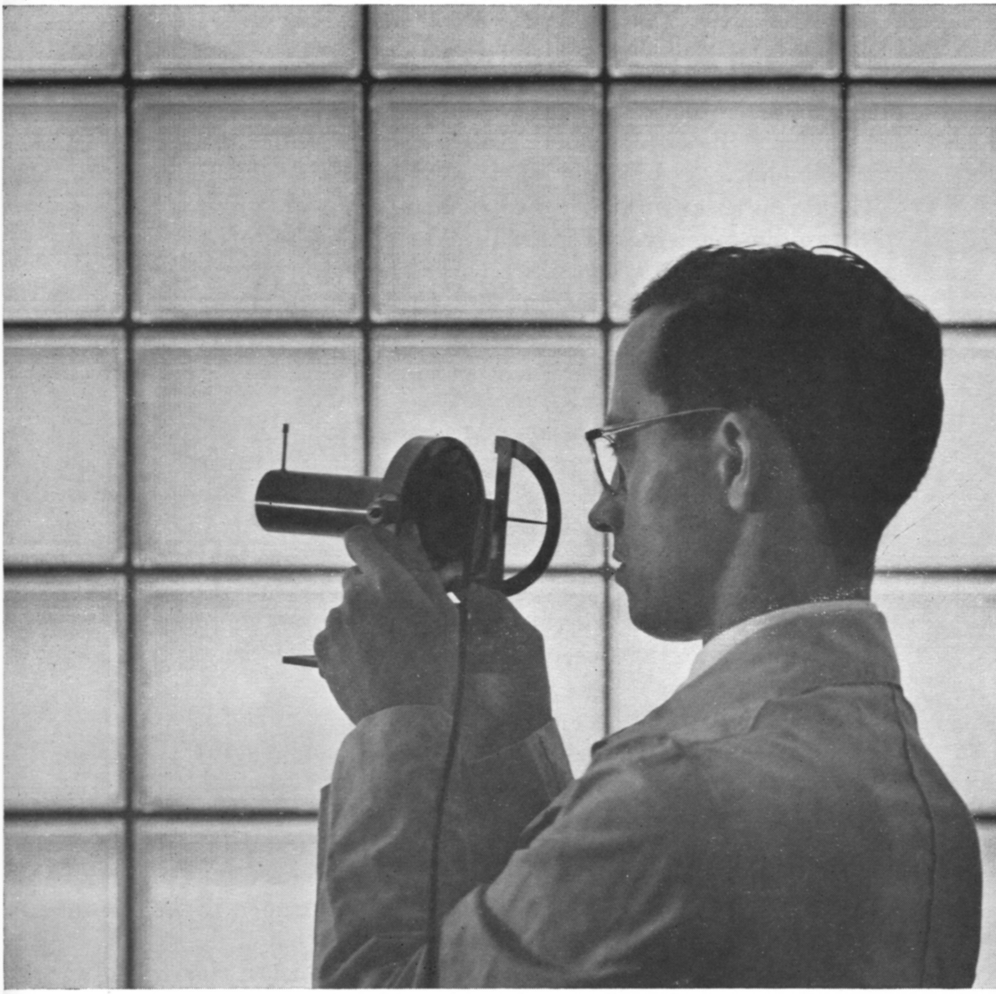


Figure 20. A direct-reading brightness meter, developed in the Laboratory, is generally used for the measurement of panel and sky brightness.

through the window. However, the light that is reflected from the earth and surroundings to the fenestration is important and should be taken into account if an accurate measurement job is to be done. The measurement of the illumination of an exterior horizontal surface appears to be a still less accurate control in that the light from the surroundings is not included and the unimportant light reflected from the exterior surface of the fenestration may be included.

In all the measurements of daylighting made by this Laboratory the exterior vertical surface illumination of the fenestration wall has been included. This appears to be the most reasonable and accurate procedure since this is the surface that is receiving light from the sun, sky, and the surroundings and is transmitting it to the interior of the building.

The brightnesses within the experimental classroom are measured with two different types of instruments: a Luckiesh-Taylor Brightness Meter (see Fig. 19) and a direct-reading brightness meter developed by this Laboratory (see Fig. 20). The Luckiesh-Taylor Brightness Meter is a visual-comparison instrument, and in the hands of an experienced operator this meter gives fairly accurate results when the surface viewed has a grey, uniform appearance; however, when the surface has a distinct color or a nonuniform brightness, it is difficult to read, and, consequently, results are inconsistent.

In order to be able to measure average brightnesses with greater accuracy, the direct-reading instrument, consisting of a barrier-layer photocell and a collimating tube, was constructed. The field of view of this instrument is such that an area of 4 square feet is included at a distance of 20 feet. The instrument can be connected to a microammeter or to the Speedomax Recorder for measurement of brightness.

The reflectivity of every surface in a room has an influence upon the quality and quantity of the daylighting. The illumination of any surface in the room, as mentioned previously, can be considered in two parts: that due to light coming directly from the fenestration and that due to light reaching the surface after one or more reflections from other surfaces. The amount of light transmitted to the room interior by the fenestration is not dependent upon the reflectivities of the room surfaces, and yet the illumination of the task is much greater when the room is decorated in light colors than when it is decorated in dark colors. It is obvious, then, that the increase of illumination is due to the increase of scattered light that results from the higher reflectivities. In the case of a room that is fenestrated with light-directing glass block, a high reflectivity for the ceiling and the upper portions of the walls is, of course, very essential since most of the light transmitted by the block is first incident on these surfaces.

The reflectivities of the lower room surfaces also aid in increasing the illumination and its uniformity throughout the room; but, possibly of more importance is the effect they have on the quality of the lighting. Generally, if one is interested in keeping brightness ratios

Figure 21. An integrating-sphere type of reflectometer, designed and constructed at the University, is used for all measurements of surface reflectivity. An auxiliary voltage control unit is used to assure accuracy of measurement.

involving room and work surfaces within certain limits, the ratio of reflectivities of these surfaces should be within the same limits. Since in most cases permanence and maintenance are also of prime importance, the finishing materials should be selected with a practical viewpoint.

All measurements of reflectivity in the Laboratory are made with the reflectometer shown in Fig. 21. This is an integrating-sphere type of instrument that was constructed in one of the University shops in order that slight modifications in design could be incorporated.

The New Type Prismatic Glass Block

The two prismatic glass-block panels that can be used for the daylighting of the light-study room consist of block (Fig. 22) that was first manufactured by the American Structural Products Company in May, 1950. This block differs from the old prismatic block in that the vertical rib structure on the outside and inside faces of the block has a narrower spacing and a greater ratio of depth to width.

As was mentioned previously, these vertical ribs on the two outside faces of the old block served to diffuse the transmitted light, this diffusion being a sidewise scattering of the light, since the ribs are arranged vertically. Actually, as was realized during the first years of work on the project, the ribs on the outside face can serve a different purpose than those on the inside face, and both sets can be made to exercise greater control over the transmitted light if the ribs are properly designed.

The light that reaches the outside face of the block directly from the sun and sky can come from any direction above the horizontal, and this direction can be designated in terms of an altitude angle and an azimuth angle. The horizontal prisms on the two inside faces of the block change the direction of the light as far as altitude is concerned, but they have only a small effect on its azimuth or sidewise direction. Thus, when the azimuth angle of the sun with respect to the block face is other than zero some of the light entering the cavity of the block will be lost by striking the side wall unless its direction is changed by the outside vertical ribs. Also, part of the inside half of the block will be quite dark since it is in the shadow of one side wall.

A properly designed vertical rib structure on the exterior face, then, can correct the azimuth angle of the light entering the block. This has two results: the inside face of the block is uniformly lighted from side to side, and the total amount of light transmitted by the block is greatly increased. This rib on the new prismatic block is so designed that light striking it from a wide angle is transmitted by the near side of the rib and is internally reflected by the far side of the same rib, thus causing a much greater change in direction than was possible with the old design.

The vertical rib structure on the inside face of the block has a somewhat different function in that it horizontally diffuses the light and thus causes a reduction in the head-on brightness of the block. Theoretically, at least, this rib structure can be designed to give the block any desired horizontal variation of brightness. For the new prismatic block, the head-on brightness is considerably below that of

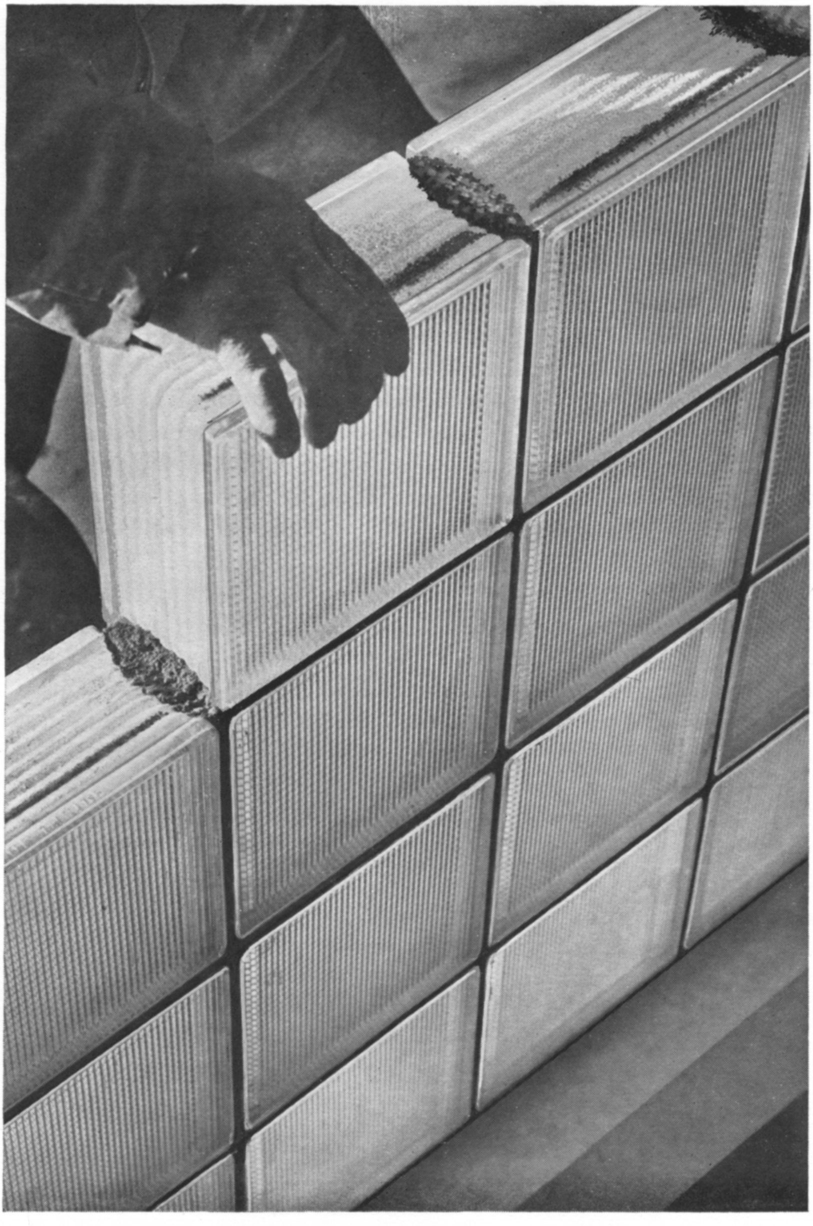


Figure 22. The new type prismatic glass block has been provided with a water-repellent face finish which prevents the adhesion of mortar and, thus, allows faster cleaning without the use of strong acids; "TOP" and "INSIDE" are indicated by a distinctive gold-colored stripe for assurance of proper orientation.

the old block, the brightness for an observer azimuth of 45 degrees is about the same, and the wide-angle brightness is substantially greater.

In general, the new prismatic block, on sun exposure, will transmit more light than the old block when the sun is at a wide angle to the panel and thus provides more uniform daylighting throughout the day.

Block brightnesses are usually quoted as average values, that is, the total amount of light transmitted in a particular direction is assigned to the entire face of the block. Experience has indicated that one prefers an average brightness that is due to a large number of low brightness sources rather than a small number of high brightness sources. The pleasing appearance of the new block is partly due to the size of the exterior rib structure; the alternate dark and light vertical strips of the face are so close together that one's eye cannot resolve them from a reasonable viewing distance. Also, the fact that the face is uniformly lighted from side to side adds to this pleasing appearance.

The old prismatic glass block was characterized by a very intense "up" beam, that is, a beam of light emerging from the block in a direction slightly above the horizontal. The directions of these beams for various exterior daylighting conditions were given previously. The widths of these beams, however, were not very great, due to the fact that the face ribs of the block caused only a small amount of horizontal diffusion. The new prismatic block, since it has the same interior prism arrangement, has "up" beams with the same elevations as those of the old block; but, with the greater horizontal diffusion caused by the rib structures, the widths of these beams are considerably greater. It is understandable that, if the new exterior face ribs cause a reduction in the head-on brightness for the directions below the horizontal by diffusion, they will do the same for the brightnesses above the horizontal. This greater diffusion of the light as it emerges from the block is an aid to the scattering of the light throughout the room and consequently an aid to the quality of the daylighting.

Performance Data

Measurements of illumination and brightness have been made in the experimental classroom shown by Fig. 12. Certain of these data will be presented here in order to show the quality and quantity of the daylighting obtained.

The classroom to which these data pertain has a single fenestration on south exposure, the block panel consisting of the new type

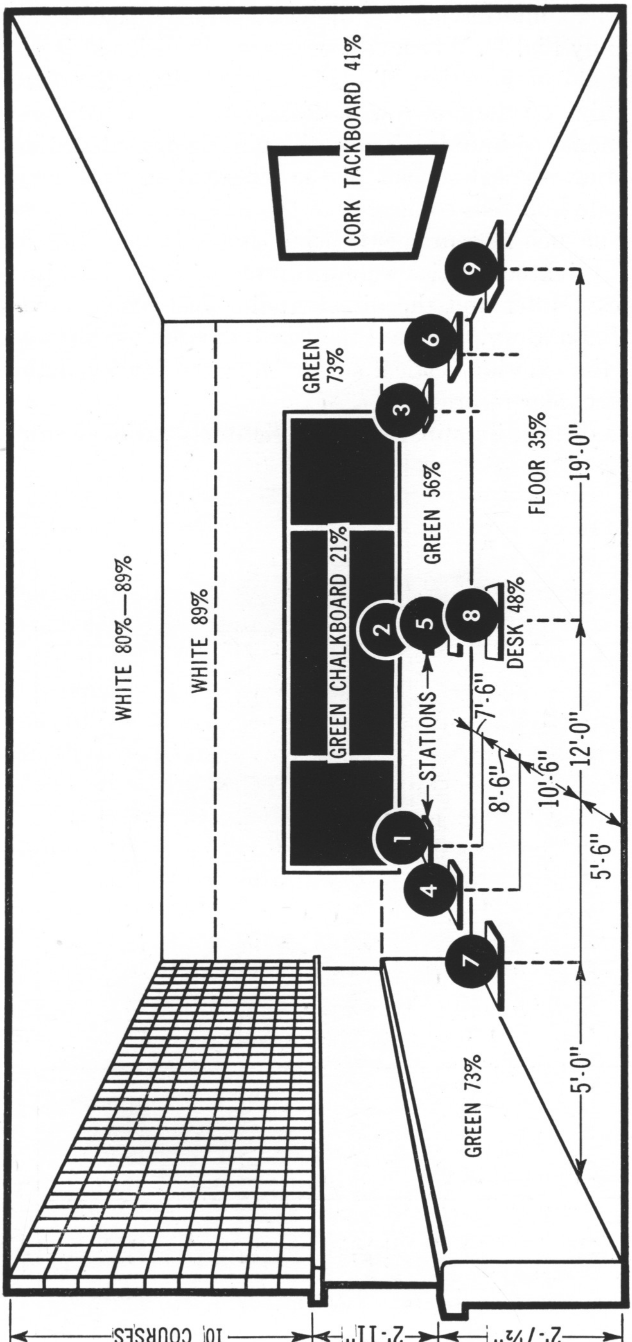


Figure 23. The classroom, see Fig. 12, for which daylighting data is given has a panel of the new type prismatic block on south exposure and the room dimensions, surface reflectivities and reference stations as indicated above. Venetian blinds were used on the vision strip to prevent direct sunlight from reaching the first row of desks.

prismatic glass block and having the dimensions and surface reflectivities indicated by Fig. 23. Whenever necessary, the vision strip was shaded with the aid of Venetian blinds in order to keep the direct sunlight from falling on the first row of desks.

The measurements of illumination at the nine indicated stations and of the exterior vertical surface of the fenestration were made with the corrected photocells connected to the Speedomax Recorder, a typical record of such measurements being shown by Fig. 30. The brightnesses and brightness ratios were measured with the Luckiesh-Taylor Brightness Meter and the direct-reading instrument developed by this Laboratory. These brightness measurements were associated with the exterior vertical surface illumination by timing them with the Recorder record.

Measurements of task illumination and brightness ratios are presented as follows:

Figure	Date	Time of Day	Exterior Illumination	Vision-Strip Shading
24	10/18/50	1:30 P.M.	Sun and Clear Sky v.s.i.*—9000 ft-c	100%
25	10/18/50	1:20 P.M.	Sun and Clear Sky v.s.i.—9000 ft-c	0%
26	11/ 1/50	9:00 A.M.	Sun and Clear Sky v.s.i.—5100 ft-c	100%
27	11/ 1/50	12:05 P.M.	Sun and Clear Sky v.s.i.—9600 ft-c	50%
28	10/22/50	11:50 A.M.	Overcast Sky v.s.i.—1000 ft-c	0%

* Vertical surface illumination.

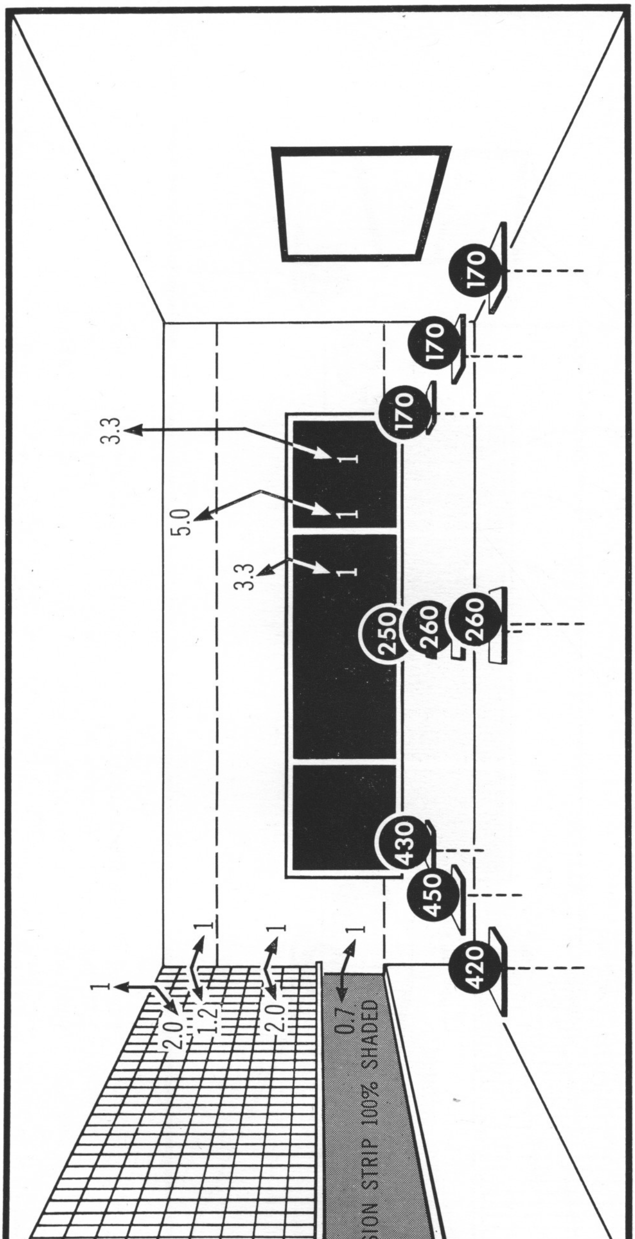


Figure 24. Task illumination, in foot-candles, and brightness ratios for 1:30 P.M. E.S.T., October 18, 1950, with sun and clear sky: V.S.I.—9000 foot-candles; panel brightness, -20° alt., 0° az.—1900 foot-lamberts,-20° alt., 45° az.—1100 foot-lamberts; brightness ratios for station 9, task to desk—2.0 to 1, task to floor—2.9 to 1, task to dado—1.1 to 1.

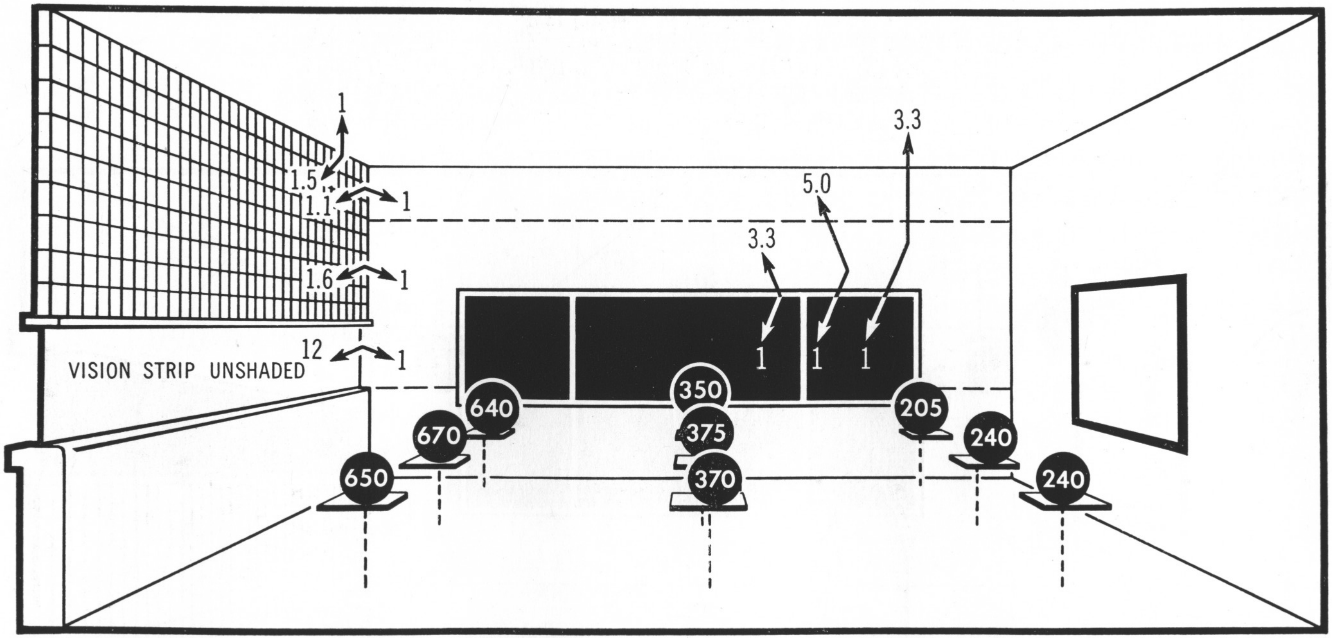


Figure 25. Task illumination, in foot-candles, and brightness ratios for 1:20 P.M. E.S.T., October 18, 1950, with sun and clear sky: V.S.I.—9000 foot-candles; panel brightness, -20° alt., 0° az.—1900 foot-lamberts -20° alt., 45° az. —1100 foot-lamberts; brightness ratios for station 9, task to desk—1.9 to 1, task to floor—2.8 to 1, task to dado—1.2 to 1.

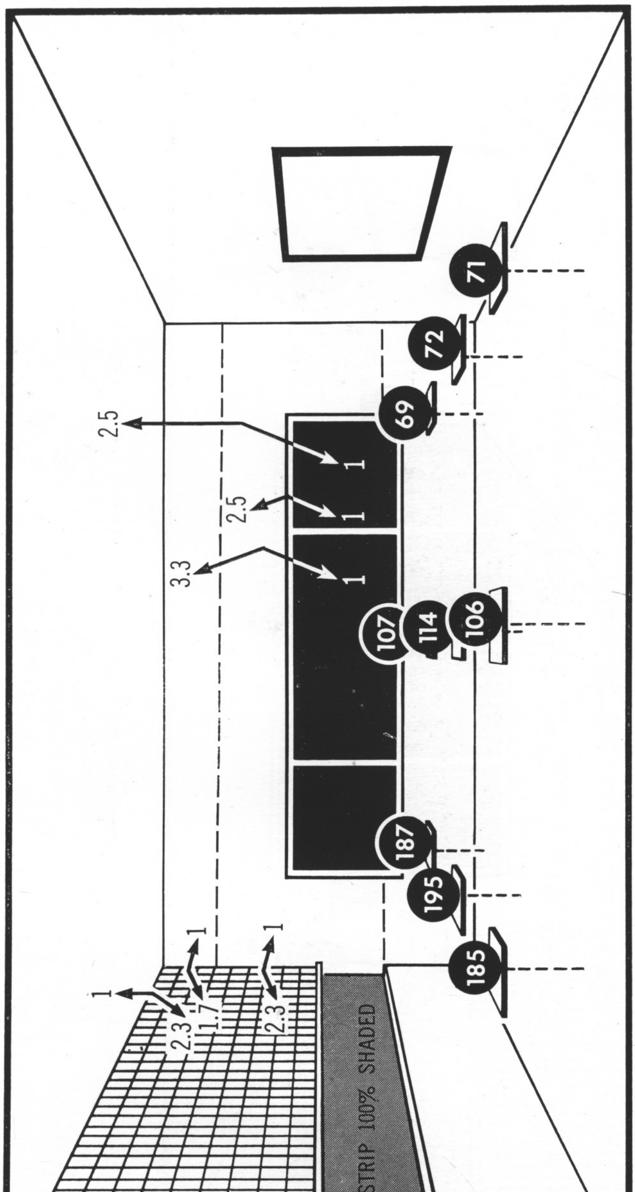


Figure 26. Task illumination, in foot-candles, and brightness ratios for 9:00 A.M. E.S.T., November 1, 1950, with sun and clear sky: V.S.I.—5100 foot-candles; panel brightness, -20° alt., 0° az.—770 foot-lamberts, -20° alt., 45° az.—460 foot-lamberts: brightness ratios for station 9, task to desk—2.0 to 1, task to floor—2.0 to 1, task to dado—1.4 to 1.

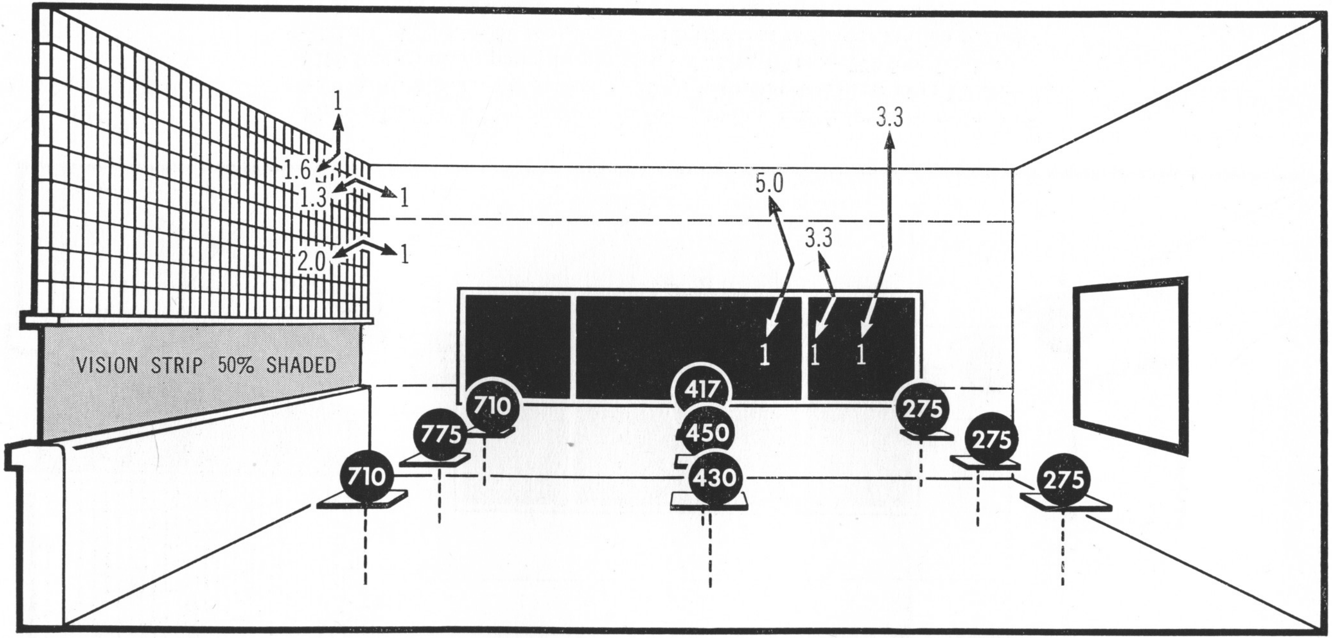


Figure 27. Task illumination, in foot-candles, and brightness ratios for 12:05 P.M. E.S.T., November 1, 1950, with sun and clear sky: V.S.I.—9600 foot-candles; panel brightness, -20° alt., 0° az.—2200 foot-lamberts, -20° alt., 45° az.—1250 foot-lamberts; brightness ratios for station 9, task to desk—1.7 to 1, task to floor—2.8 to 1, task to dado—1.6 to 1.

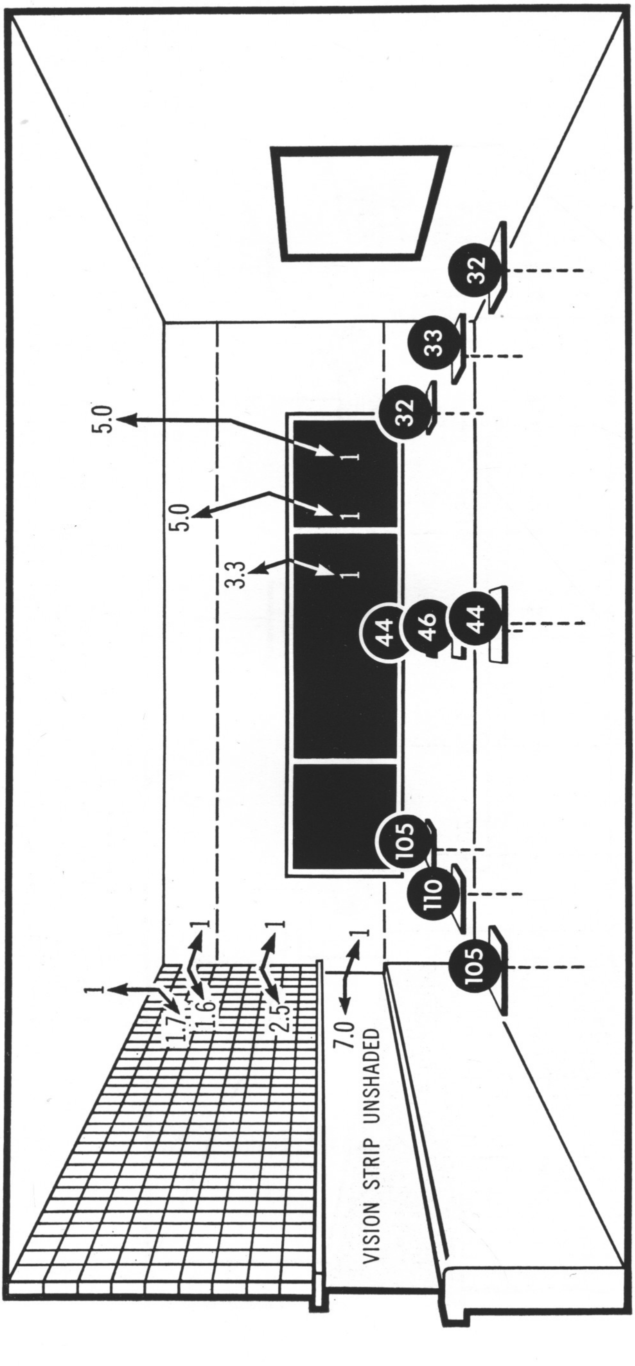


Figure 28. Task illumination, in foot-candles, and brightness ratios for October 22, 1950, with overcast sky: V.S.I.—1000 foot-candles; brightness ratios for station 9, task to desk—2.0 to 1, task to floor—2.9 to 1, task to dado—1.2 to 1.

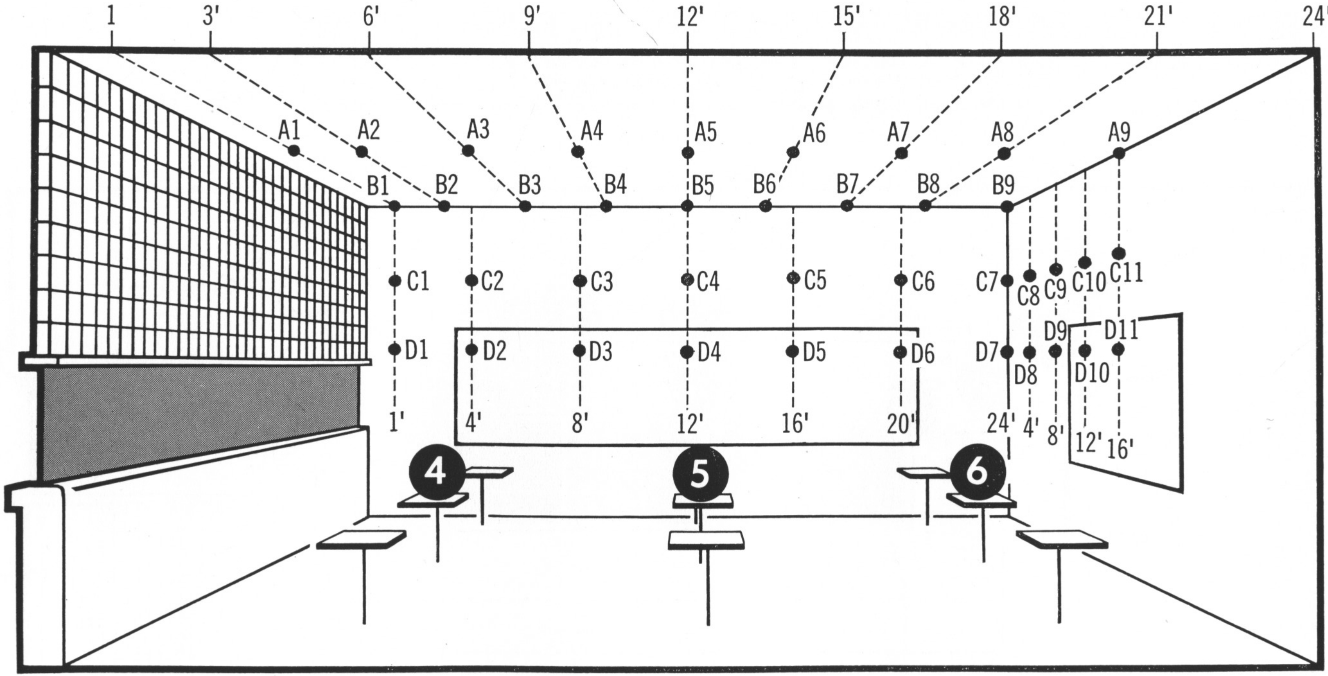


Figure 29. The table on the opposite page gives the illumination on each of the classroom surfaces, indicated by the above diagram, for a V.S.I. of 9400 foot-candles due to sun and clear sky between 12:35 P.M. and 1:20 P.M. on November 1, 1950. The vision strip was completely shaded in order to determine the distribution of daylight throughout the room due to the action of the prismatic block.

In order to show the distribution of light about a classroom, a photocell was used with the Speedomax Recorder to collect the results as indicated by Fig. 29 and the following table. The vision strip was completely shaded and the illumination values were measured between 12:35 P.M. and 1:20 P.M. on November 1, 1950. The distribution of light is given for only one-half of the room; the task illuminations are given only along the centerline of the room. During the time that the measurements were being taken the exterior vertical surface illumination varied from 9600 to 9200 foot-candles, and, consequently, all results have been scaled to 9400 foot-candles.

An analysis of these data shows that of the 230 foot-candles of illumination at station 6 only 23 per cent is due to light coming directly from the glass-block panel, whereas 53 per cent is due to light diffusely reflected from the ceiling. These figures serve to indicate the action of the prismatic block in transmitting light to the ceiling for diffuse reflection and, consequently, the importance of the ceiling in establishing a higher illumination at the more remote stations in the room.

Room Illumination in Foot-Candles

A1—1170	B1—1100	C1 —1060	D1 —630
A2—1000	B2— 830	C2 — 720	D2 —450
A3— 710	B3— 580	C3 — 440	D3 —340
A4— 590	B4— 400	C4 — 320	D4 —270
A5— 350	B5— 300	C5 — 260	D5 —220
A6— 230	B6— 220	C6 — 210	D6 —190
A7— 150	B7— 190	C7 — 280	D7 —250
A8— 160	B8— 170	C8 — 300	D8 —270
A9— 190	B9— 180	C9 — 320	D9 —290
Station 4—630		C10— 320	D10—360
Station 5—380		C11— 320	D11—300
Station 6—230			

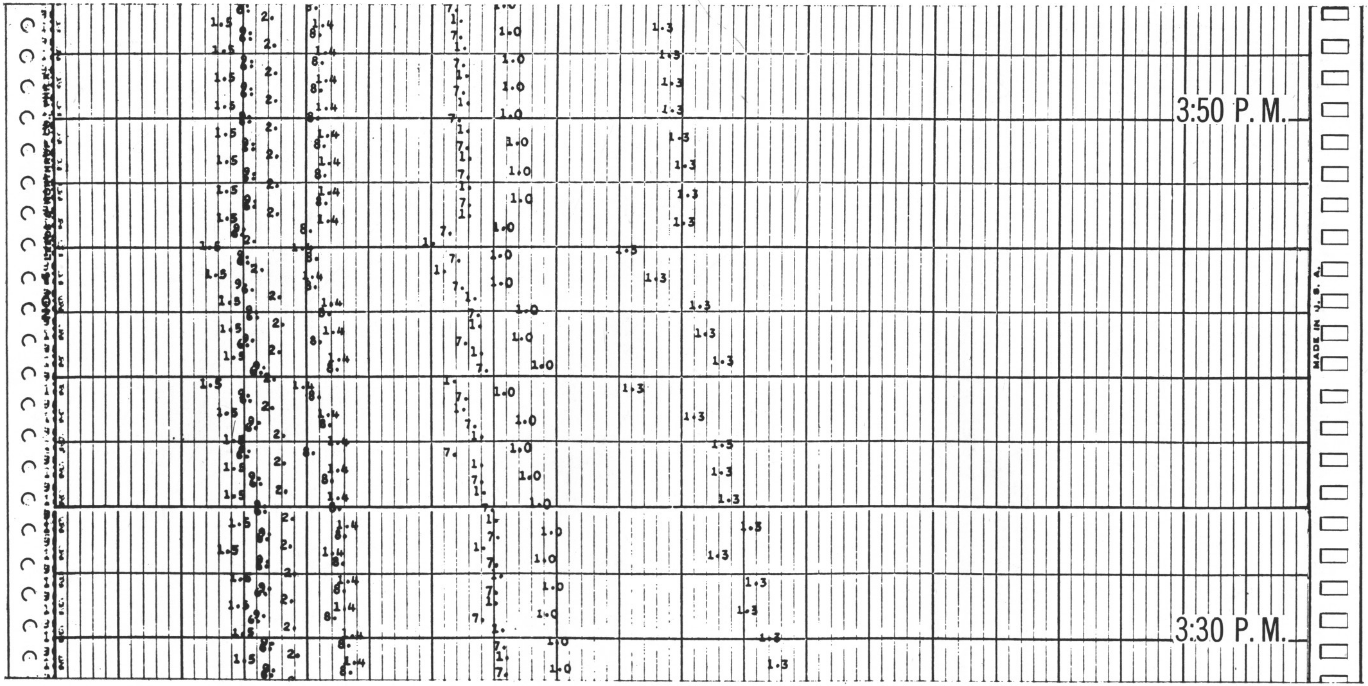


Figure 30. The ten series of points indicate the illumination at the nine classroom stations, shown by Fig. 23, and of the panel exterior. The displacement of the point from the left-hand base line multiplied by a calibration constant gives the illumination in foot-candles.

Discussion of Performance Data

It is interesting to compare the quality and quantity of the daylighting in this classroom with those specified in the *Recommended Practice of Daylighting* for school classrooms.

The quantity of illumination recommended is 30 foot-candles or more. In this experimental classroom 33 foot-candles is obtained for a task remote from the fenestration on an overcast day when the exterior vertical surface illumination is 1000 foot-candles. When direct sunlight falls on the fenestration, the illumination of this task is well above 30 foot-candles even with the vision strip completely shaded at 9:00 A.M. on a winter day.

For a comparison of quality of the illumination consider the following table:

Brightness Ratios Between:	Recommended	Experimental Room	
		Overcast Day	Sun and Clear Sky
Task and desk	3 to 1	2.0 to 1	2.0 to 1
Task and floor	10 to 1	2.9 to 1	2.8 to 1
Task and wall dado	10 to 1	1.2 to 1	1.6 to 1
Chalkboard and ceiling	1 to 10	1 to 5.0	1 to 3.3
Panel and adjacent wall	20 to 1	1.5 to 1	1.7 to 1
Vision strip and adjacent wall	20 to 1	7.0 to 1	12.0 to 1

One will note that, exclusive of the ratio of vision strip to adjacent wall, these ratios are just about the same for overcast-sky and for sun and clear-sky conditions. This is not surprising considering that with prismatic control of the daylight the distribution of the light throughout the room is about the same for all conditions. It is understandable that the brightness ratio involving the vision strip changes, since without control one can view a changeable bright sky and consequently encounter very high brightness ratios. However, with this type of fenestration, the vision-strip brightness can be controlled, and the quality and quantity of the daylighting throughout the room can be mainted at very acceptable levels.

The diversity of the daylighting, i.e., the ratio of maximum illumination to minimum, in this room is low. It ranges from 2.7 to 3.0, depending upon the shading of the vision strip. Such a low diversity is to be compared to 5 or greater for a similar room fenestrated with clear sheet glass[1].

Appendix A
Illumination From Surface Sources

In considering the problem of daylighting from an analytical point of view, one is confronted with sources of light having two dimensions that are comparable to those of the interior itself; i.e., the sources are surface sources. In such cases the inverse square law of illumination cannot be applied directly but must be included in an integration which covers the entire area of the source.

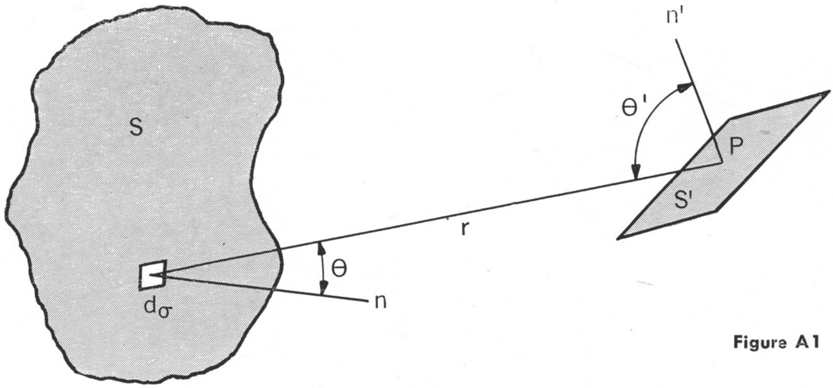


Figure A1

In Fig. A1 consider that a surface S′ is receiving light from a source S. The illumination at point P is given by the basic equation,

$$E_p = \frac{1}{\pi}\int_S \frac{B}{r^2} \cos\theta \cos\theta' \, d\sigma \qquad \text{A1.1}$$

in which $d\sigma$ is an element of area of the source to which the inverse square law can be applied, θ and θ' are the angles indicated, r is the distance from the element $d\sigma$ to the point P, and B is the brightness* of the element $d\sigma$ in the direction θ. The brightness B may be a known function or it may be a constant. If B is expressed in foot-lamberts** and r is expressed in feet, E_p will be in lumens per square foot or foot-candles.

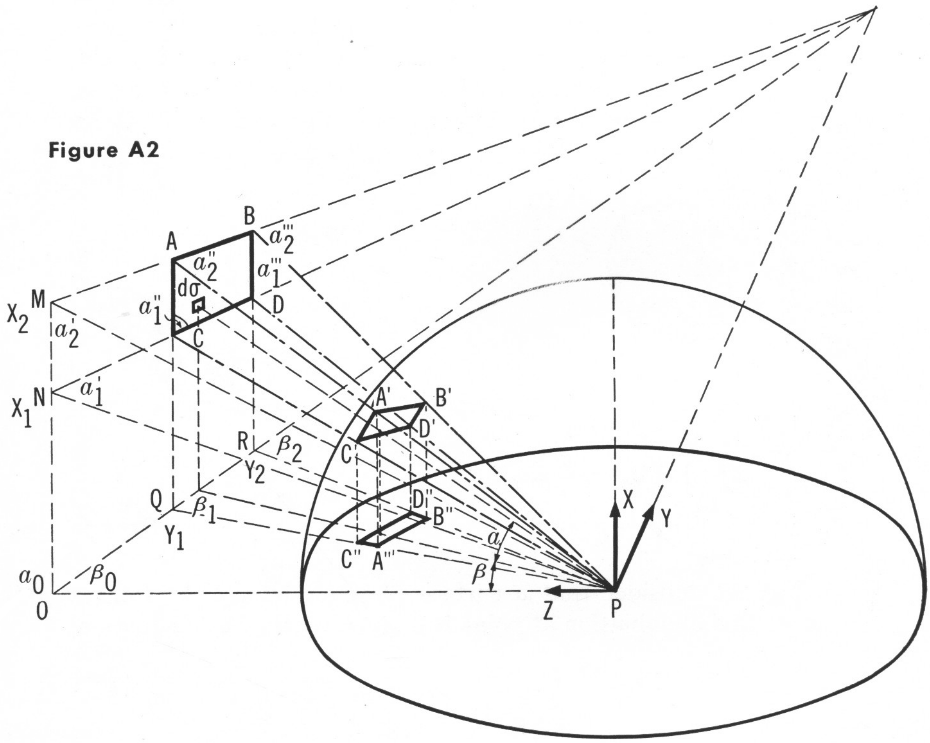
*Brightness is the luminous intensity of any surface in a given direction per unit of projected area of the surface as viewed from that direction.

**The foot-lambert is a unit of brightness equal to $1/\pi$ candle per square foot, or to the uniform brightness of a perfectly diffusing surface emitting or reflecting light at the rate of one lumen per square foot, or to the average brightness of any surface emitting or reflecting light at that rate. The average brightness of any reflecting surface in foot-lamberts is therefore the product of the illumination in foot-candles by the reflection factor of the surface.

The Rectangular Source

The rectangular source is encountered most frequently in daylighting problems since it represents a fenestration that is contributing light to a surface that is parallel or at right angles to the plane of the source.

Figure A2



In Fig. A2 consider that the surface source ABCD is emitting light and that an expression is required for the illumination at the point P. If rectangular coordinates are oriented as indicated, with the origin at P, then E_x and E_z will represent the illumination at point P for surfaces that are perpendicular and parallel, respectively, to the plane of ABCD. In applying Eq A1.1 to this case it is customary to consider that the source ABCD is a perfect diffuser and, therefore, that B is a constant. Also,

$$
\begin{aligned}
d\sigma &= dy\,dx \\
r^2 &= x^2 + y^2 + z^2 \\
\cos\theta &= z/r \\
\cos\theta' &= z/r \text{ for } E_z \text{ and} \\
\cos\theta' &= x/r \text{ for } E_x.
\end{aligned}
$$

After substitution in Eq A1.1

$$E_x = \frac{B}{\pi} \int_{x_1}^{x_2} \int_{y_1}^{y_2} \frac{zx}{r^4}\, dy\, dx \qquad A1.2$$

and

$$E_z = \frac{B}{\pi} \int_{x_1}^{x_2} \int_{y_1}^{y_2} \frac{z^2}{r^4}\, dy\, dx. \qquad A1.3$$

The integration of these equations, using rectangular coordinates, results in expressions that are lengthy and cumbersome to handle. Because of this consider the coordinates α and β as indicated by Fig. A2. It will be seen that the two coordinate systems are connected by

$$\tan \beta = y/z \qquad A1.4$$

and

$$\tan \alpha = \frac{x}{\sqrt{y^2 + z^2}} = x/z \cos \beta. \qquad A1.5$$

After substitutions of Eqs A1.4 and A1.5 in Eqs A1.2 and A1.3, one has

$$E_x = \frac{B}{\pi} \int_{\alpha_1}^{\alpha_2} \int_{\beta_1}^{\beta_2} \sin \alpha \cos \alpha\, d\beta\, d\alpha \qquad A1.6$$

and

$$E_z = \frac{B}{\pi} \int_{\alpha_1}^{\alpha_2} \int_{\beta_1}^{\beta_2} \cos^2 \alpha \cos \beta\, d\beta\, d\alpha. \qquad A1.7$$

The limits α_1 and α_2 are functions of β, as follows:

$$\alpha_1 = \tan^{-1} \frac{x_1}{z} \cos \beta = \tan^{-1} \tan \alpha_1' \cos \beta \qquad A1.8$$

$$\alpha_2 = \tan^{-1} \frac{x_2}{z} \cos \beta = \tan^{-1} \tan \alpha_2' \cos \beta \qquad A1.9$$

After integration of Eqs A1.6 and A1.7, substitution of limits, and collection of terms, one has

$$\begin{aligned} E_x = \frac{B}{2\pi} & (\cos \alpha_1' \tan^{-1} \cos \alpha_1' \tan \beta_2 \\ & - \cos \alpha_2' \tan^{-1} \cos \alpha_2' \tan \beta_2 \\ & - \cos \alpha_1' \tan^{-1} \cos \alpha_1' \tan \beta_1 \\ & + \cos \alpha_2' \tan^{-1} \cos \alpha_2' \tan \beta_1) \end{aligned} \qquad A1.10$$

and

$$\begin{aligned} E_z = \frac{B}{2\pi} & (\sin \alpha_2' \tan^{-1} \cos \alpha_2' \tan \beta_2 \\ & + \sin \beta_2 \tan^{-1} \tan \alpha_2' \cos \beta_2 \\ & - \sin \alpha_1' \tan^{-1} \cos \alpha_1' \tan \beta_2 \\ & - \sin \beta_2 \tan^{-1} \tan \alpha_1' \cos \beta_2 \\ & - \sin \alpha_2' \tan^{-1} \cos \alpha_2' \tan \beta_1 \\ & - \sin \beta_1 \tan^{-1} \tan \alpha_2' \cos \beta_1 \\ & + \sin \alpha_1' \tan^{-1} \cos \alpha_1' \tan \beta_1 \\ & + \sin \beta_1 \tan^{-1} \tan \alpha_1' \cos \beta_1). \end{aligned} \qquad \text{A1.11}$$

The interpretation of Eqs A1.10 and A1.11 becomes much easier if one considers the entire area OMBR to be the surface source; then

$$\alpha_1' = \beta_1 = 0$$

and

$$E_x = \frac{B}{2\pi} (\beta_2 - \cos \alpha_2' \tan^{-1} \cos \alpha_2' \tan \beta_2) \qquad \text{A1.12}$$

and

$$\begin{aligned} E_z = \frac{B}{2\pi} & (\sin \alpha_2' \tan^{-1} \cos \alpha_2' \tan \beta_2 \\ & + \sin \beta_2 \tan^{-1} \tan \alpha_2' \cos \beta_2). \end{aligned} \qquad \text{A1.13}$$

By considering Eqs A1.12 and A1.13 in relation to Eqs A1.10 and A1.11, respectively, one observes that

$$\begin{aligned} E_x\,(ABCD) &= E_x\,(OMBR) - E_x\,(ONDR) \\ &\quad - E_x\,(OMAQ) + E_x\,(ONCQ) \qquad \text{A1.14} \\ E_z\,(ABCD) &= E_z\,(OMBR) - E_z\,(ONDR) \\ &\quad - E_z\,(OMAQ) + E_z\,(ONCQ). \qquad \text{A1.15} \end{aligned}$$

Since

$$\tan^{-1} \cos \alpha_2' \tan \beta_2 = \beta_2'''$$

and

$$\tan^{-1} \tan \alpha_2' \cos \beta_2 = \alpha_2''',$$

where β_2''' is the angle BPM and α_2''' is the angle BPR, Eqs A1.12 and A1.13 are further simplified to be

$$E_x = \frac{B}{2\pi} (\beta_2 - \beta_2''' \cos \alpha_2') \qquad \text{A1.16}$$

and

$$E_z = \frac{B}{2\pi} (\beta_2''' \sin \alpha_2' + \alpha_2''' \sin \beta_2). \qquad \text{A1.17}$$

By using Eqs A1.16 and A1.17, which give the illumination at the point P when P is on the normal at one corner of a rectangular surface source, one can evaluate each of the four terms on the right side of Eqs A1.14 and A1.15 and thereby determine E_x (ABCD) and E_z (ABCD) without using the more cumbersome equations.

Graphical Representation

A graphical method that is useful in daylighting problems for determining the importance of walls and ceilings as sources of diffuse illumination without laborious calculation will be presented in this section.

In Fig. A2 consider the hemisphere that is shown as having its base on the yz plane (working plane) and its center of curvature at the point P. The projection of the surface source ABCD onto the surface of this hemisphere as A′B′C′D′ can be considered as the source of light having a brightness B. An intermediate step in the integration of Eq A1.6 is

$$E_x = \frac{B}{2\pi}\Big[\cos\alpha_1' \tan^{-1}\cos\alpha_1' \tan\beta - \cos\alpha_2' \tan^{-1}\cos\alpha_2' \tan\beta\Big]_{\beta_1}^{\beta_2} \qquad \text{A2.1}$$

and, therefore, specifies the illumination at P due to area A′B′C′D′.

In addition consider that the area A′B′C′D′ is projected normally onto the base of the hemisphere and that this projected area is A″B″C″D″. An element of area in A′B′C′D′ is represented by

$$da' = R^2 \cos\alpha \, d\beta \, d\alpha$$

and an element in A″B″C″D″ by

$$da'' = R^2 \cos\alpha \sin\alpha \, d\beta \, d\alpha.$$

If, however, the area on the base of the hemisphere is expressed in polar coordinates, β and $r = R\cos\alpha$,

then

$$da'' = -r \, d\beta \, dr$$

and

$$a'' = \int_{r_1}^{r_2}\int_{\beta_1}^{\beta_2} -r \, d\beta \, dr, \qquad \text{A2.2}$$

where, from Eqs A1.8 and A1.9,

$$r_1 = \frac{R}{\sqrt{1 + \tan^2 \alpha_1' \cos^2 \beta}}$$

and

$$r_2 = \frac{R}{\sqrt{1 + \tan^2 \alpha_2' \cos^2 \beta}}.$$

After integration of Eq A2.2 and the substitution of limits r_1 and r_2

$$a'' = \frac{R^2}{2}\Big[\cos \alpha_1' \tan^{-1} \cos \alpha_1' \tan \beta - \cos \alpha_2' \tan^{-1} \cos \alpha_2' \tan \beta\Big]_{\beta_1}^{\beta_2}. \qquad \text{A2.3}$$

By comparing Eq A2.3 with Eq A2.1, one observes that

$$E_x = \frac{Ba''}{\pi R^2}, \qquad \text{A2.4}$$

which indicates that the illumination, in foot-candles, at point P on a plane perpendicular to the plane of the source ABCD is given by the product of the brightness, in foot-lamberts, and the ratio of the projected area a″ to the total area of the base of the arbitrary hemisphere. Furthermore, one can observe that Eq A2.4 does not contain any of the coordinates that were used to determine the location and orientation of the source. Thus, the projected area and the surface brightness determine the illumination due to any source. When the source is a hemisphere having a uniform brightness of B foot-lamberts, the illumination at the point P is B foot-candles. This same result is obtainable from Eq A1.1.

Fig. A3 is a graphical representation, in accordance with the above equations, of the room surfaces shown by Fig. A4, the projection being for the station that is on the center-line of the room and 19 feet from the wall containing the area ABCD. The dotted lines extending to the points V and V are for a room that extends to infinity in two directions.

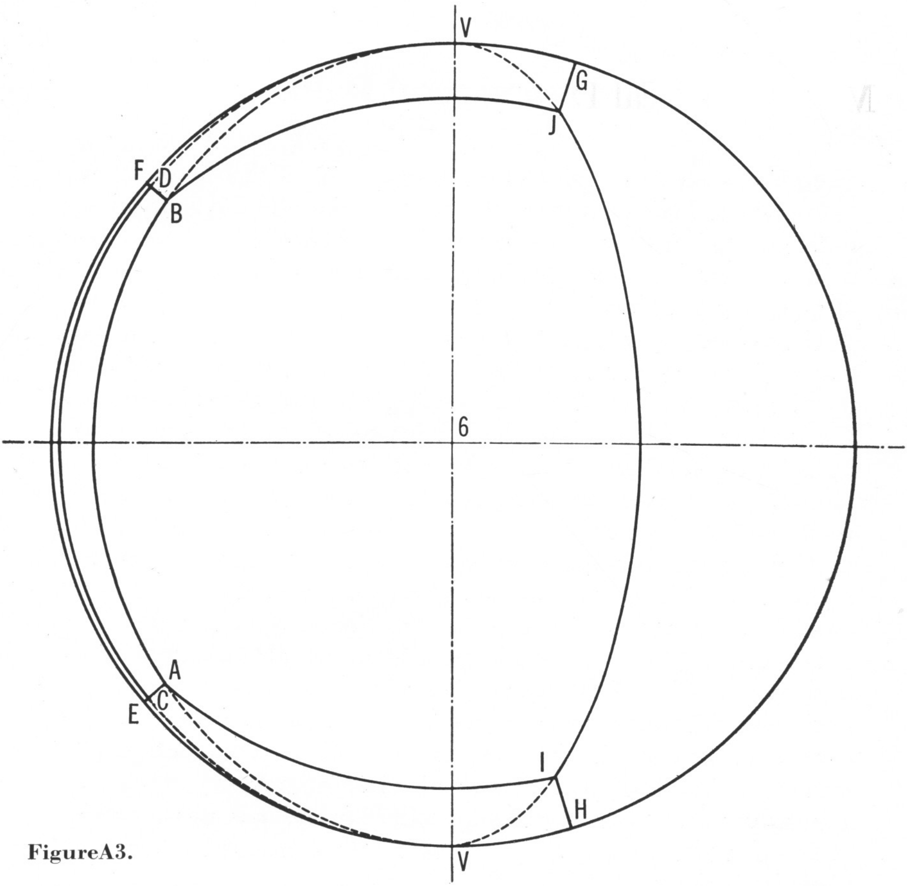


Figure A3.

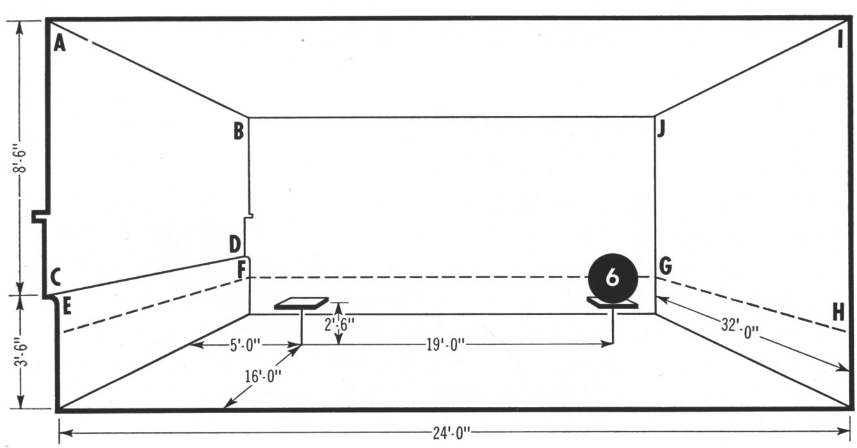


Figure A4.

Appendix B

Meteorological Data

The purpose of this section is to present some of the meteorological data and methods of calculating sun positions and daylight intensities, collected from various papers (2–9 and 13-14) that have been used in determining the brightness and transmission characteristics of prismatic glass block.

The primary source of all daylight is, of course, the sun, and the major secondary source is the sky. Even though the sun follows definite paths with respect to a specific portion of the earth's surface, the brightness of the sun, the brightness of the sky, and, consequently, the total amount of light reaching the surface of the earth are dependent upon the composition of the atmosphere and the position of the sun. In addition, the total amount of daylight falling on any particular building surface is dependent upon the position and extent of the cloud formations and the arrangement and reflectivities of the surrounding earth, vegetation, and buildings.

Because of these many variables it is not possible to calculate the total amount of daylight at a particular surface at any given moment. Such quantities must be measured with suitable instruments. What can be determined is the average or probable illumination at a particular time and place, and this is often of greater value in daylighting problems than the results obtained from single measurements.

The position of the sun with respect to a particular horizontal plane is given by

$$\sin \alpha = \sin \phi \sin \sigma + \cos 15t \cos \phi \cos \sigma \qquad \text{B1.1}$$

and

$$\sin \beta = \frac{\cos \sigma \sin 15t}{\cos \alpha}, \qquad \text{B1.2}$$

where the five angles are defined as follows:

α = the angular distance (altitude) between the sun and the horizon as measured along the great circle which passes perpendicular to the plane of the horizon, through the sun, and through the zenith. It is measured positively from the horizon to the zenith, from 0° to 90°.

β = the angle (azimuth) between the vertical plane containing the sun and the plane of the meridian. Angles measured west of south are considered positive and angles east of south negative.

ϕ = the latitude of the place. North latitude is positive.

σ = the angular position (declination) of the sun at true noon with respect to the equator.

t = the time, in hours, measured from noon; time before noon being negative and time after noon positive.

In using these equations one must have, in addition to the latitude and longitude of the place, the declination of the sun and the equation of time. These latter quantities will be found in the larger almanacs or in the literature of the subject, but for completeness here, they are given in Table B1 for a number of days in the year (7). The equations, as stated above, give mean sun time, and the corrections, as given by the equation of time, are made to this mean sun time to get the actual time of the event as read on a clock. Since standard time is in general use, this actual sun time must be corrected to agree with the prime meridian of the standard time zone of the place in question. If the location for which the computation is being made is west of the prime meridian of the standard time zone, a correction of four minutes of time must be added for each degree westward of the standard time meridian, and four minutes subtracted for each degree eastward (8).

The intensity of direct sunlight at the surface of the earth can be calculated in accordance with the laws of absorption. Accordingly, this intensity on a plane at right angles to the direction of the sun is

$$I = I_o \, 10^{-\epsilon M}, \qquad \text{B1.3}$$

where I_o designates the intensity of sunlight at the boundary of the atmosphere, and ϵ is the absorption coefficient of the stratum of air of thickness M.

Values of the air mass M, as calculated by Bemporad (9) for different solar altitudes, are given in Table B2.

Various authors working with published meteorological data have evaluated I_o and ϵ of Eq B1.3, and, accordingly, Benford (8) gives

$$E_N = 12{,}560 \; 10^{-0.1M} \qquad \text{B1.4}$$

as the illumination in foot-candles on a plane at right angles to the direction of the sun.

These values represent the maximum illumination from the sun that can be expected at any time. Due to the variations in the amount of dirt and water vapor in the atmosphere, the probable intensities will be much less.

The horizontal illumination, in foot-candles, at the surface of the earth due to light from a clear sky can be calculated with fairly good approximation by

$$E' = 1500 \sqrt{\sin \alpha}\ ,$$

where α is the solar altitude (7).

The equations given above for the calculation of illumination intensities have been included in this section since they have been used occasionally in the work on prismatic glass block. However, Kimball and Hand (2, 3) have published considerable meteorological data that are based on direct measurements and which, therefore, have had greater application to this work. Some of these data are presented herewith.

Table B3 presents the altitude and Table B4 the azimuth of the sun at one-hour intervals for various times of the year at 42° N. latitude. Table B5 presents the solar illumination intensity at normal incidence for the central plains at 42° N. latitude for the same time of the day and year. Similar data for 30° N., 36° N., and 48° N. latitude are also given by Kimball and Hand.

In considering the light that comes from the sky it is common practice to consider two types, a cloudless sky and a completely cloudy sky. For a cloudless sky the brightness of a portion of the sky varies with its position with respect to the sun. In general, the sky is brighter near the sun and near the horizon and less bright as these distances increase. Brightness distributions for clear sky with sun at altitudes of 0°, 20°, 40°, and 60° are presented in Figs. B1, B2, B3, and B4. These figures are stereographic projections of the half of the sky on either side of the sun's vertical. The sun's position is indicated by the letter X. The horizontal straight line represents the horizon, and above it are lines of equal altitude, 10° apart. Extending from the zenith to the horizon are azimuth circles, also 10° apart. The brightness of each region of the sky is expressed in millilamberts (1 millilambert = 0.929 foot-lambert) and in ratio to the zenith brightness.

Fig. B5 is a similar graph for the brightness of an overcast or completely cloudy sky.

It is obvious that the illumination of a vertical surface due to light from a clear sky is dependent upon the direction in which the surface faces. Vertical and horizontal surface illuminations due to clear and cloudy skies are listed in Table B6. These data show the dependence of vertical and horizontal illumination upon the orientation of the surface and upon the altitude of the sun.

Figs. B6 and B7 show the dependence of illumination intensities upon solar altitude for winter and summer months. By interpolating between the curves it is possible to determine the illumination intensity for both summer and winter conditions on a vertical surface facing any desired azimuth from the sun and with the sun at any specific altitude.

Fig. B8 shows the average and minimum values of horizontal surface illumination due to cloudy sky for the Plains States and the Atlantic Coast States. With these curves and known solar altitudes one may determine the average and minimum illumination that may be expected with an overcast sky at any hour of the day at different latitudes.

TABLE B1

DECLINATION OF THE SUN AND EQUATION OF TIME

Date		Declination °	Equation of Time	Date		Declination °	Equation of Time
January	1	−23.1°	+ 3^m	July	9	+22.5°	+ 5^m
	10	−22.1	+ 7		19	+21.0	+ 6
	20	−20.3	+11		29	+18.9	+ 6
	30	−17.8	+13	August	8	+16.4	+ 6
February	9	−14.9	+14		18	+13.3	+ 4
	19	−11.5	+14		28	+10.0	+ 1
March	1	− 7.8	+13	September	7	+ 6.3	− 2
	11	− 4.0	+10		17	+ 2.5	− 5
	21	0.0	+ 7		27	− 1.4	− 9
	31	+ 3.9	+ 4	October	7	− 5.2	−12
April	10	+ 7.7	+ 2		17	− 9.0	−14
	20	+11.3	− 1		27	−12.6	−16
	30	+14.6	− 3	November	6	−15.8	−16
May	10	+17.5	− 4		16	−18.6	−15
	20	+19.8	− 4		26	−20.8	−13
	30	+21.7	− 3	December	6	−22.4	− 9
June	9	+22.9	− 1		16	−23.3	− 5
	22	+23.5	+ 2		23	−23.5	− 1
	30	+23.2	+ 3		30	−23.2	+ 2

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TABLE B2

AIR MASS, M, CORRESPONDING TO DIFFERENT SOLAR ALTITUDES

(According to Bemporad)

Solar Altitude	0	1°	2°	3°	4°	5°	6°	7°	8°	9°
0°	....	26.96	19.79	15.36	12.44	10.40	8.90	7.77	6.88	6.18
10°	5.60	5.12	4.72	4.37	4.08	3.82	3.59	3.39	3.21	3.05
20°	2.90	2.77	2.65	2.55	2.45	2.36	2.27	2.20	2.12	2.06
30°	2.00	1.94	1.88	1.83	1.78	1.74	1.70	1.66	1.62	1.59
40°	1.55	1.52	1.49	1.46	1.44	1.41	1.39	1.37	1.34	1.32
50°	1.30	1.28	1.27	1.25	1.24	1.22	1.20	1.19	1.18	1.17
60°	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.09	1.08	1.07
70°	1.06	1.06	1.05	1.05	1.04	1.04	1.03	1.03	1.02	1.02
80°	1.02	1.01	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00
90°	1.00	....	....	....	....	....	....	....	....	....

TABLE B3
SOLAR ALTITUDES
(Latitude 42° N)

Date	Sun's Hour Angle from Meridian							
	0	1	2	3	4	5	6	7
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
December 21	24 33	23 06	18 55	12 28	4 17	.....	.....	.....
January 21	28 06	26 34	22 11	15 27	6 59	.....	.....	.....
February 21	37 29	35 43	30 42	23 14	14 05	3 50	.....	.....
March 21	48 16	46 08	40 18	31 56	22 02	11 17	0 11	.....
April 21	59 53	57 09	50 06	40 40	30 05	19 01	7 55	.....
May 21	68 12	64 47	56 37	46 24	35 26	24 18	13 22	2 54
June 21	71 27	67 41	59 04	48 35	37 31	26 25	15 34	5 14
July 21	68 29	65 03	56 50	46 36	35 37	24 29	13 33	3 06
August 21	60 07	57 23	50 21	40 52	30 15	19 11	8 05	.....
September 21	48 42	46 31	40 42	32 18	22 21	11 35	0 28	.....
October 21	37 17	35 29	30 32	23 04	13 55	3 42	.....	.....
November 21	28 04	26 32	22 09	15 23	6 58	.....	.....	.....

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TABLE B4
SOLAR AZIMUTHS
(Latitude 42° N)

Date	Sun's Hour Angle from Meridian									
	Declination	0	1	2	3	4	5	6	7	Sunrise or Sunset
	° ′		°	°	°	°	°	°	°	°
December 21	−23 27	0	15	29	42	53	..	...	...	57
January 21	−19 54	0	17	30	44	55	..	...	...	62
February 21	−10 31	0	18	35	50	62	72	...	...	75
March 21	0 16	0	22	41	56	69	80	90	...	91
April 21	11 53	0	28	50	66	78	89	99	...	107
May 21	20 12	0	35	58	74	86	96	105	115	119
June 21	23 27	0	39	63	78	89	99	108	117	123
July 21	20 29	0	35	59	75	86	96	106	115	119
August 21	12 07	0	28	50	66	79	89	99	...	107
September 21	0 42	0	22	41	56	69	80	91	...	92
October 21	−10 43	0	18	35	49	61	72	...	...	75
November 21	−19 56	0	17	30	44	55	..	...	...	62

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TABLE B5
SOLAR ILLUMINATION INTENSITY AT NORMAL INCIDENCE
(Latitude 42° N. Central Plains)

Date		Sun's Hour Angle from the Meridian Measured by Foot-Candles							
		0	1	2	3	4	5	6	7
December	21	7,040	6,850	6,150	4,850	2,120	....	....	....
January	21	7,540	7,320	6,790	5,540	3,200	....	....	....
February	21	8,500	8,350	7,780	6,860	5,170	1,940	....	....
March	21	9,140	8,990	8,520	7,740	6,460	4,320	490	....
April	21	9,150	8,980	8,550	7,820	6,860	5,290	3,110	....
May	21	9,100	8,920	8,500	7,760	6,890	5,480	3,700	1,110
June	21	9,180	9,020	8,590	7,920	6,970	5,750	4,060	2,120
July	21	9,030	8,870	8,450	7,770	6,830	5,490	3,670	1,120
August	21	8,710	8,540	8,130	7,360	6,320	4,750	2,740	....
September	21	8,550	8,390	7,880	7,030	5,700	3,560	460	....
October	21	8,010	7,870	7,450	6,480	4,720	1,770	....	....
November	21	7,520	7,320	6,730	5,440	3,250	....	....	....

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TABLE B6
ILLUMINATION FROM SKYLIGHT, WASHINGTON, D. C.

Solar Altitude	On Horizontal Surface	ON VERTICAL SURFACE — Azimuth between normal to surface and sun's azimuth Foot-Candles							Zenith Brightness mL	
		0°	45°	70°	90°	135°	180°	Mean		
0°	15.2	5.6	5.8	.....	6.4	6.7	7.1	6.3	15.8	Cloudy Sky
20.2°	726	298	280	.....	273	273	272	279	989	
41.0°	1,505	614	608	...	615	622	606	613	2,000	
61.4°	2,150	881	941	...	977	932	929	932	3,600	
71.4°	2,950	1,142	1,103	...	1,118	1,122	1,203	1,138	4,840	
20°	840	1,252	1,028	803	526	316	293	...	400	Clear Sky, Summer
40°	1,340	1,454	1,325	932	686	417	358	...	803	
60°	1,600	1,420	1,255	923	751	559	486	...	1,650	
70°	1,600	1,291	1,074	903	754	542	475	...	2,300	
0°	67.8	64.6	63.7	...	30.6	30.2	31.5	...	27.1	Clear Sky, Winter
20°	683	1,042	873	562	393	265	257	...	281	
40°	977	1,121	936	690	505	325	295	...	544	

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Figure B1 **Clear Sky** **ALT. 0°**

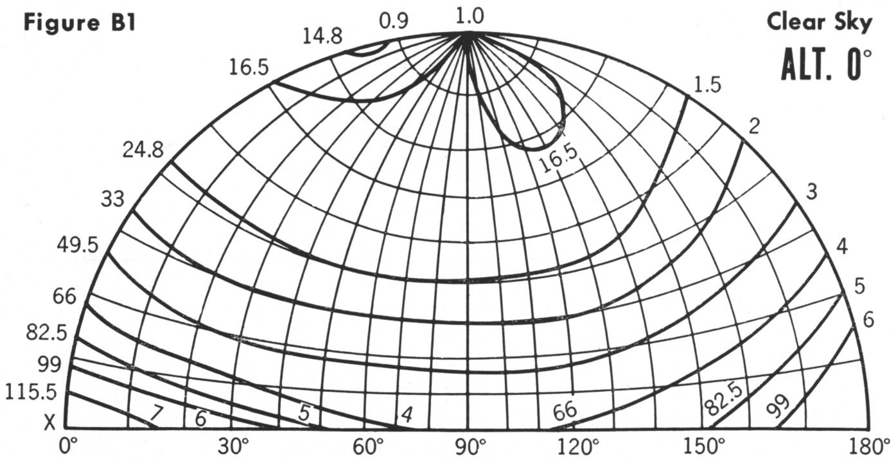


Figure B2 **Clear Sky** **ALT. 20°**

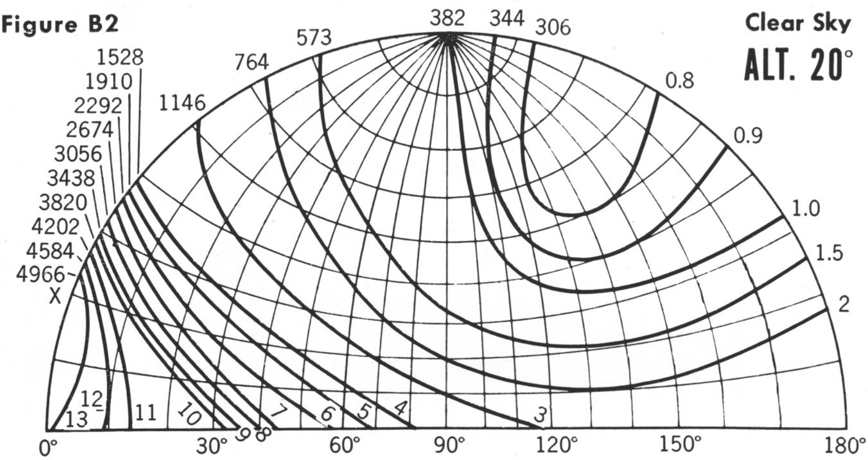


Figure B3 **Clear Sky** **ALT. 40°**

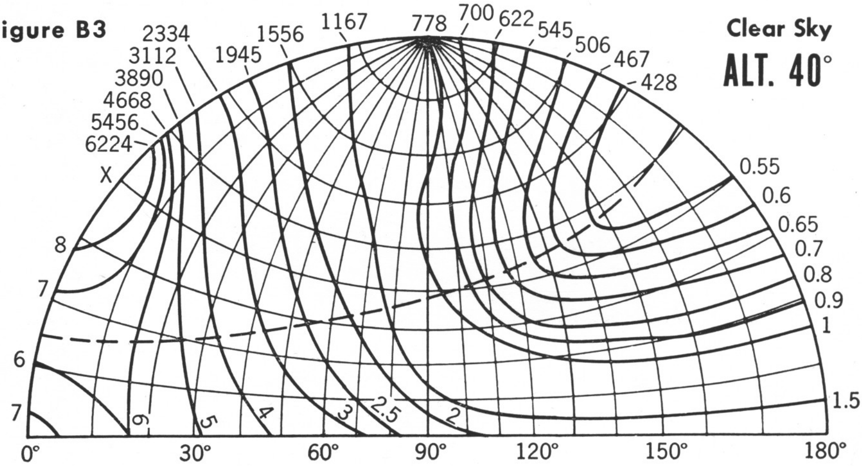


Figure B4

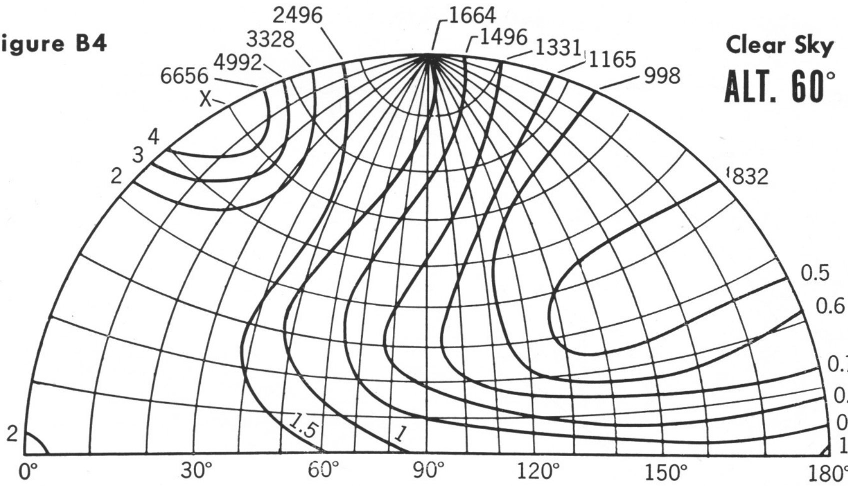
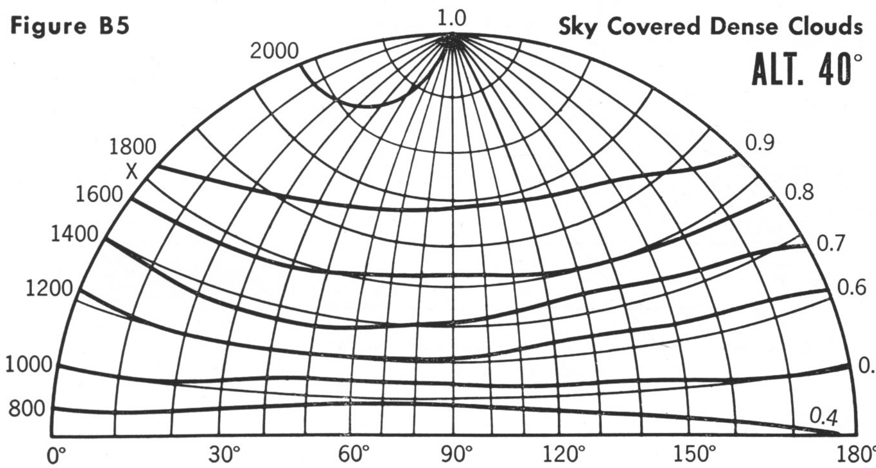


Figure B5



Figures B1, B2, B3 and B4. The mean brightness of a clear sky in millilamberts, and with reference to the zenith brightness, for solar altitudes 0°, 20°, 40°, and 60°, respectively, is shown by means of lines of equal brightness. The sun's position is indicated by the letter X. Half the sky is shown in stereographic projection. These data are based on numerous measurements of sky brightness made in Washington, D.C. between April 5 and July 14, inclusive, 1921. (Reference 13)

Figure B5. The brightness of an overcast sky, with sun 40° above the horizon and clouds so dense that neither blue sky nor the sun is visible, is presented in a similar manner. (Reference 14)

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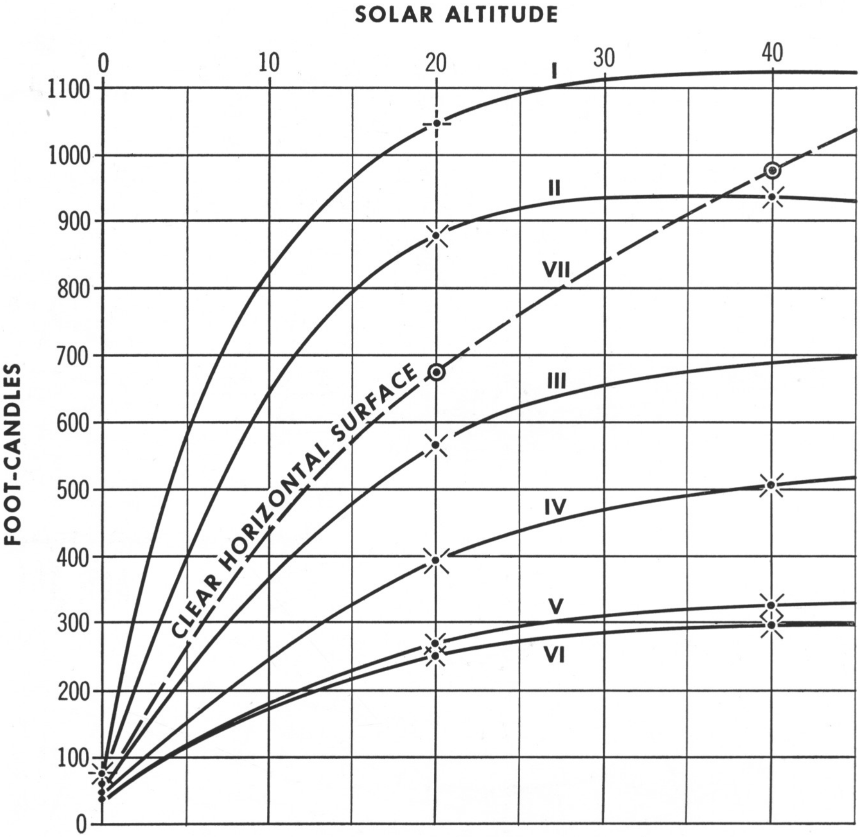
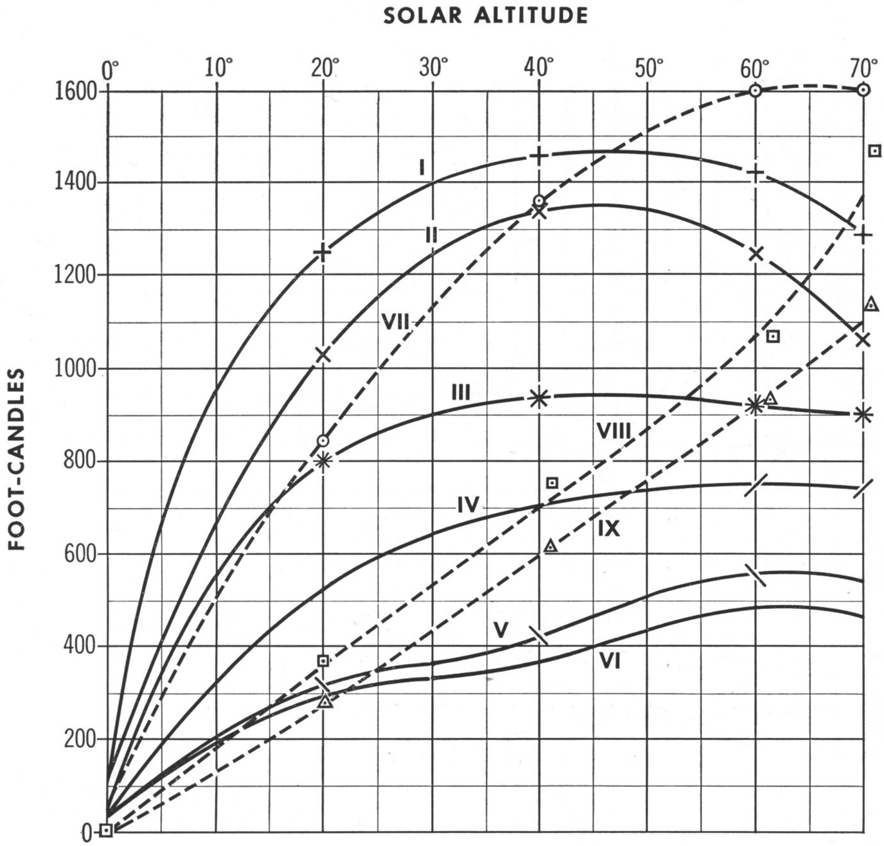


Curve I.	Clear sky. Vertical surface facing 0° in azimuth from sun.
Curve II.	Clear sky. Vertical surface facing 45° in azimuth from sun.
Curve III.	Clear sky. Vertical surface facing 70° in azimuth from sun.
Curve IV.	Clear sky. Vertical surface facing 90° in azimuth from sun.
Curve V.	Clear sky. Vertical surface facing 135° in azimuth from sun.
Curve VI.	Clear sky. Vertical surface facing 180° in azimuth from sun.
Curve VII.	Clear sky. Horizontal surface.

Fig. B6. Curves of winter skylight illumination intensity on different surfaces.

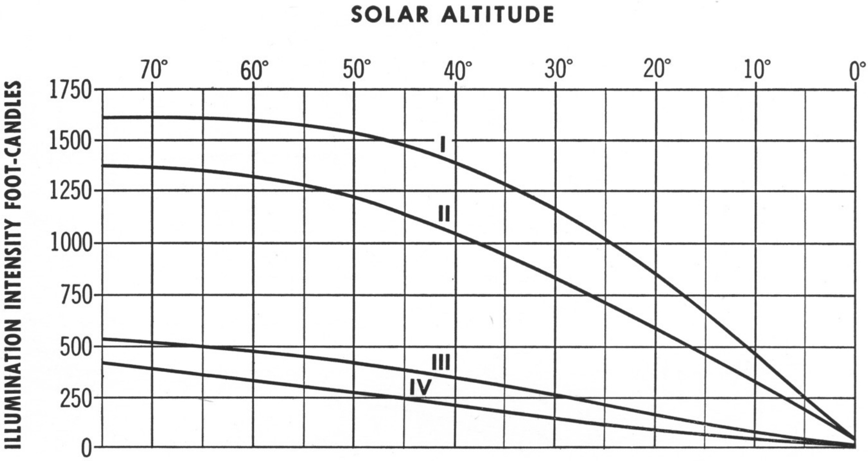
By Courtesy of Illuminating Engineering Society



Curve	I.	Clear sky. Vertical surface facing 0° in azimuth from sun.
Curve	II.	Clear sky. Vertical surface facing 45° in azimuth from sun.
Curve	III.	Clear sky. Vertical surface facing 70° in azimuth from sun.
Curve	IV.	Clear sky. Vertical surface facing 90° in azimuth from sun.
Curve	V.	Clear sky. Vertical surface facing 135° in azimuth from sun.
Curve	VI.	Clear sky. Vertical surface facing 180° in azimuth from sun.
Curve	VII.	Clear sky. Horizontal surface.
Curve	VIII.	Cloudy sky. Horizontal surface. (Intensity scale X2).
Curve	IX.	Cloudy sky. Vertical surface.

Figure B7. Curves of summer skylight illumination intensity on different surfaces.

By Courtesy of Illumination Engineering Society



Curve I.	Average for Plains States.
Curve II.	Average for Atlantic Coast States.
Curve III.	Minimum for Plains States.
Curve IV.	Minimum for Atlantic Coast States.

Figure B8. Illumination intensity with a cloudy sky.

U. S. Weather Review, 1919

Appendix C
A Photocell that Obeys the Cosine Law of Illumination and the Integrating Spere

The Photocell

The photocells that have been used in this work are the photronic cells that are manufactured by the Weston Electrical Instrument Corporation, Newark, New Jersey. In using these barrier-layer or self-generating cells one avoids the use of considerable auxiliary equipment because, at lower illumination levels, they generate an electromotive force of 2 to 4 millivolts per foot-candle. In using these cells for general illumination work, however, one must avoid several principal errors. These are as follows:

1) Errors due to differences between the spectral response curve and the standard visibility curve
2) Errors due to temperature and fatigue effects
3) Errors due to deviations from the cosine law of illumination.

These cells can be obtained with a special filter (Viscor Visual Correction Filter) which practically eliminates the errors of item 1 above. The errors of item 2 above can be reduced to a negligible minimum by the proper manipulation of the electrical constants and by the use of metallic film filters.

The errors in measurement of illumination that one may encounter when using a barrier-layer photocell that does not obey the cosine law are serious when accurate results are required. These errors are characteristic not only of the Weston cell but of other barrier-layer cells as well. After investigating several types of such cells a project was undertaken to develop an attachment for the Weston cell that would cause it to obey the cosine law of illumination. The final result is the corrected cell that is shown in Fig. 16. The accuracy and operation of this unit will be compared to those of the uncorrected photocell.

The illumination of a surface is the luminous flux which it receives per unit area. When a collimated beam of light encounters a plane surface the illumination of that surface is given by

$$I = I_o \cos \theta, \qquad \text{C1.1}$$

where θ is the angle between the direction of the beam and the normal to the surface, and I_o is the illumination that the surface would receive if $\theta = 0$. This fundamental law is the cosine law of illumination.

Eq C1.1 evaluates the illumination of the surface correctly, providing the energy arrives at the surface from only one direction. When a surface is illuminated from more than one direction, then

$$I = \sum (I_o)_i \cos \theta_i, \qquad \text{C1.2}$$

where the summation extends over all the incident energy.

The response of a photocell in a particular circuit can then be expressed as

$$R = k\, A_o\, I = k\, A_o \sum (I_o)_i \cos \theta_i, \qquad \text{C1.3}$$

where A_o is the effective area of the cell face and k is a proportionality factor. In order for the cell to obey the cosine law the product k A_o must be independent of θ.

The relative response curves of Fig. C1 show the deviation of the Weston Photronic cells, with and without a Viscor filter, from the cosine law. These curves have been so adjusted that all responses for $\theta = 0$ are equal to unity. It can be observed from these curves that the quantity k A_o decreases as θ increases and actually becomes zero at $\theta = 85°$. The variation of A_o makes up the major part of this change and this is due to the shading of the actual cell face by the rim of the cell case. The factor k, independent of A_o, also decreases as θ increases. This is due partly to the variation in the reflection of light either from the glass plate that protects the cell face or from the Viscor filter elements and partly to the action of the cell face itself.

Since the cell has circular symmetry, the deviations listed above, except for very minor changes, are independent of the azimuth position of the plane containing the incident beam and the normal to the cell face.

The curves of Fig. C2 show the percentage errors that would be encountered when the uncorrected cells are used for the measurement of unidirectional illumination. If the cells were to be used only for the measurement of unidirectional or multidirectional illumination for which all the directions of incidence are known, then sufficient accuracy of measurement could be obtained by using Eq C1.2 and the curves of Fig. C1. In general, however, one wishes to measure diffuse illumination having an unknown distribution, and therefore it is impossible to follow this procedure.

The corrected photocell, as can be seen from Fig. 16, consists of a housing that holds a barrier-layer photocell and a circular disc of flashed opal glass. The glass disc has its fritted face and edge slightly depolished. When the cell unit is to be used for the measurement of low illuminations, a spacing ring separates the disc from the cell face. For high illumination measurements a metallic film filter is substituted for this spacing ring.

The reason that the flashed opal disc in the assembled unit extends above the plane of its surroundings is that the depolished fritted surface does not in itself transmit light in accordance with the cosine law, and therefore this deficiency in transmitted energy must be compensated for by the light transmitted by the edge of the disc. The disc should not receive and transmit light from angles greater than 90°; therefore, the metal shield has an outer rim raised to the level of the top of the disc.

The degree of obeyance of the cosine law by this photocell unit is also indicated on Figs. C1 and C2. The scale of Fig. C1 is not large

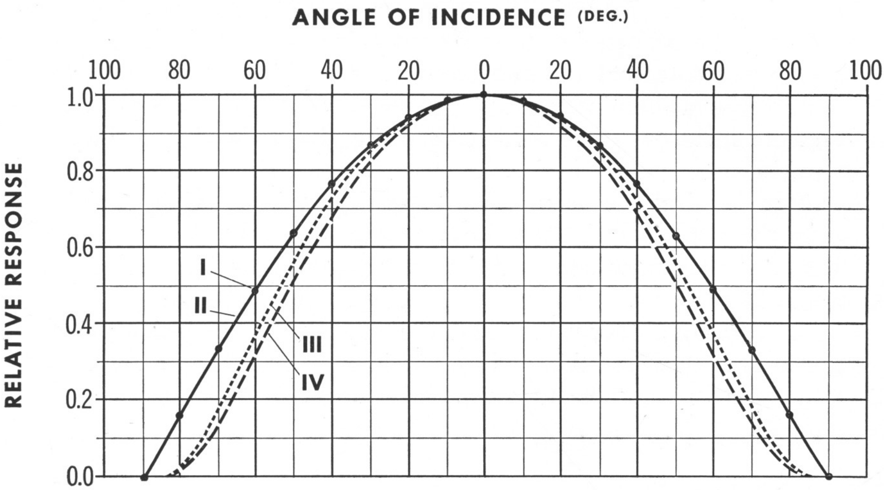


Figure C1. Relative response curves. I. Weston photronic cell—with Viscor filter and corrective attachment. II. Cosine of angle of incidence. III. Weston photronic cell—no filter. IV. Weston photronic cell—with Viscor filter.

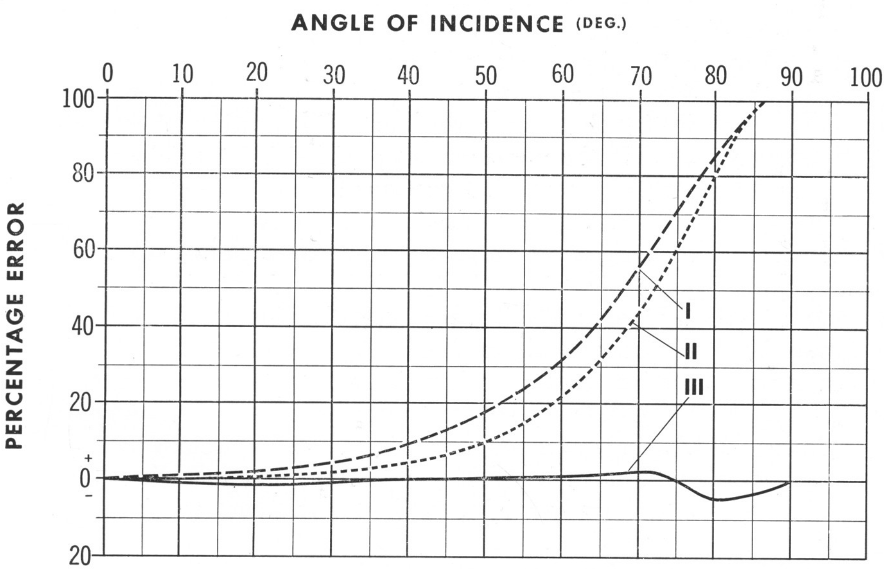


Figure C2. Percentage error curves. I. Weston photronic cell—with Viscor filter. II. Weston photronic cell—no filter. III. Weston photronic cell—with Viscor filter and corrective attachment.

enough to show the deviations from the cosine law. Hence, the percentages of error are shown by the solid-line curve of Fig. C2.

One will note that the maximum error of this corrected unit for unidirectional illumination is 4 per cent and that it occurs for an angle of incidence of 80°, this error being due primarily to the use of a shield of reasonable size. In measuring the illumination due to a uniform hemispherical source, such as a sky having a uniform brightness, this corrected photocell has an error of less than one per cent. For a similar measurement, the uncorrected cells have errors of about 20 per cent.

In other laboratory work where the directions of the incident light are known or where the angle of incidence can be made zero, the uncorrected photocells are used to advantage.

The Integrating Sphere

In recent years integrating spheres have been used rather extensively in photometric work for the measurement of the total lumens emitted by electric lamps and of reflection and transmission coefficients of certain materials. However, they have not been used appreciably for the measurement of the total amount of daylight transmitted by fenestration materials. This is due partly to the lack of application and partly to the fact that the functioning of the integrating sphere is not particularly suited to the measurement of light that is transmitted by nonfunctional fenestration materials.

In the work that was anticipated with functional glass block, however, it was felt that a properly adapted integrating sphere would be very valuable in the measurement of the light transmitted from natural and artificial sources. Consequently, the integrating sphere shown by Figs. 8 and 10 was constructed, and it has been used extensively in the laboratory work, as will be described hereafter.

An integrating sphere is a spherical shell that has its inside surface coated with a highly reflecting, perfectly diffusing, and nonselective material. With these conditions it can be proved that every element of surface illuminates every other element to the same degree no matter how the wall of the sphere or any portion of it is illuminated directly (10). Thus, if the surface of the sphere has a uniform reflection factor, and if a small portion of it is made luminous, the whole sphere will be uniformly bright, except for this spot which may be of negligible proportions.

With the above in mind, consider the theory as follows:

Assume that an area of A_o square feet of the inside surface of the sphere is replaced by a material that is transmitting L lumens and that A_c square feet is replaced by a barrier-layer photocell that may or may not have its view of the remainder of the sphere wall restricted.

The light received by the cell can be considered in three parts, namely: (1) the amount that comes directly from the transmitter or area A_o, (2) the amount that comes from the sphere wall after one reflection, and (3) the amount that is the result of multiple reflections. Consider each of these three parts separately:

(1) If the transmitting area were a perfect diffuser, then its brightness in any direction would be L/A_o foot-lamberts. Since this is never the case, its brightness in the direction of the photocell is Lk/A_o, where k is a proportionality factor dependent on angle. The intensity of the illumination at the face of the photocell is

$$I_1 = \frac{Lk}{A}, \qquad \text{C1.4}$$

where $A = 4\pi R^2$, the total area of the inside surface of the sphere.

(2) The L lumens transmitted through the area A_o falls on the inside surface of the sphere, but it may not be uniformly distributed since the transmitting material has a specific brightness pattern. Therefore assume that αL lumens falls on area a and $(1\text{-}\alpha)$ L lumens falls on area (A-a). However, although the sphere coating is not a perfect diffuser, it is a good diffuser, and therefore, the brightness of these areas can be represented as $\frac{\alpha L}{a} r_s k'$ and $\frac{(1-\alpha)L}{(A-a)} r_s k''$, where r_s is the reflectivity of the sphere coating and k' and k'' are factors only slightly different from unity. The intensity of illumination at the face of the photocell due to these two components is

$$I_2 = \frac{\alpha L r_s k'}{A} + \frac{(1-\alpha) L r_s k''}{A}. \qquad \text{C1.5}$$

(3) The intensity of illumination at the face of the photocell due to the remaining reflections is

$$I_3 = \frac{L r_a^2}{A} + \frac{L r_a^3}{A} + \text{----} + \frac{L r_a^n}{A}, \qquad \text{C1.6}$$

where r_a is the average reflectivity of the sphere wall.

Now, if the photocell obeys the cosine law of illumination, these three parts can be added to form the total photocell response, i.e.,

$$R = I_1 + I_2 + I_3$$
$$= \frac{Lk}{A} + \frac{\alpha L r_s k'}{A} + \frac{(1-\alpha) L r_s k''}{A}$$
$$+ \frac{L r_a^2}{A} + \frac{L r_a^3}{A} + \cdots + \frac{L r_a^n}{A}$$

$$= \frac{L}{A} (k + \alpha r_s k' + (1-\alpha) r_s k'' + r_a^2 + r_a^3 + \cdots + r_a^n). \quad \text{C1.7}$$

One observes that R can be used as a measure of the total lumens admitted to the sphere interior, provided the lack of perfect diffusion of the emitter and of the sphere coating do not cause the quantity within the brackets to differ appreciably from its value for the case of perfect diffusion. For the case of perfect diffusion k, k′, and k″ are all unity, $r_a = r_s$, and the bracketed quantity, when n is infinite, has a value of

$$\frac{1}{1 - r_s}.$$

The factor k, since it indicates the brightness of the area A_o in the direction of the photocell, could be very large. Therefore, in designing the sphere, it was considered advisable to place the photocell so that it could not see the area A_o, i.e., $k = 0$.

The areas A_o and A_c are the only portions of the inside surface that may have reflectivities other than r_s. In the sphere $A_c = 0.087$ square feet, and the maximum value for A_o is 1.000 square foot, whereas A is 113 square feet. Thus, regardless of the reflectivity of A_c and A_o, the value of r_a will be greater than 0.99 r_s.

Applying these limitations to Eq C1.7, gives

$$R = \frac{L}{A} \left(\frac{r_a}{1-r_a}\right), \quad \text{C1.8}$$

providing $k' = k'' = 1$. Now, as far as the sphere coating is concerned, k′ and k″ are only slightly different from unity; hence, the only way the components $r_s k'$ and $(1-\alpha) r_s k''$ can cause appreciable error is for them to fail to reach the photocell from some locations. This is not possible since the photocell can see the entire sphere wall except A_o, A_c, and a small area immediately surrounding them.

In actual use in the Laboratory the integrating sphere is calibrated by using an incandescent lamp in the sphere for which the total lumen output is known, by admitting a collimated beam of light of known intensity through an unglazed opening, and by admitting light through a diffusing plate for which total transmission is known.

Appendix D

The Determination and Use of Brightness and Transmission Data for Prismatic Glass Block

As mentioned previously, the collection of brightness and transmission data for prismatic glass block cannot be made with natural sources without an appreciable loss of time and accuracy. Consequently, the artificial sources and control equipment were constructed and methods of measurement developed in order to obtain greater efficiency of operation and acceptable accuracy.

These data for direct sunlight and various sky conditions must be collected in sufficient detail and in such a form that they can be used in the prediction of block performance.

The purpose of this Appendix is to present the methods of measurement used with the artificial sources and a brief outline of the use of these data in the prediction of daylighting.

The Determination of Block Brightness and Transmission for Direct Sunlight

In Fig. D1 consider that the artificial sun projects a collimated beam of light onto the exterior face of the block from a direction specified by the altitude angle α' and the azimuth angle β', the two angles referring to the center of the exterior face of the block. Also consider that the photocell on the hoop faces the block from a direction specified by the altitude angle α and the azimuth angle β, the two angles referring to the center of the interior face of the block.

The only light reaching the photocell comes directly from the block face since all the surrounding surfaces are black and the photocell is equipped with a shield that restricts its view. The intensity of the illumination on the photocell can be expressed by Eq A1.1 of Appendix A as follows:

$$E_p = \frac{1}{\pi}\int_S \frac{B}{r^2} \cos\theta \cos\theta' \, d\sigma$$

becomes

$$I\theta = \frac{B\theta \, A_b}{\pi R^2} \cos\alpha \cos\beta, \qquad \text{D1.1}$$

since $\cos\theta = \cos\alpha \cos\beta$, $\cos\theta' = 1$ (θ' always zero), and R (a constant) is the radius of the hoop. I_θ is the intensity on the hoop photocell, in foot-candles, due to the area A_b at a distance R, having an average brightness of B_θ foot-lamberts in a direction α, β from its center.

This photocell, then, measures the average brightness of the block in a specific direction for a particular illumination of its exterior. In making such measurements for a large number of directions, however, the exterior illumination must remain constant, or else a circuit must be used that prevents the variations from affecting the results.

The circuit of Fig. D1 is a bridge arrangement in which a second photocell is used in order to obtain a null reading on the galvanometer G; i.e., the potential drop across R_1 due to the action of the hoop cell C_1 is made equal and opposite to the potential drop across ΔR due to the action of the fixed cell C_2. The cell C_2 receives its light through an adjustable filter from a fixed lamp that is connected to the voltage supply of the artificial sun. If this fixed lamp is properly selected, then the percentage change of its lumen output, due to small changes in voltage and frequency of the power supply, will be the same as that of the artificial sun, and the bridge setting ΔR remains constant.

The exterior cell resistances R and $R_1 + R_2$ are selected so that the circuit currents i and i_θ are directly proportional to the intensities I and I_θ over the range of measurements to be made. Thus

$$i = \frac{k\,I}{r + R}$$

and

$$i_\theta = \frac{k' I_\theta}{r' + R_1 + R_2},$$

where k, k′, r, and r′ are circuit constants. Then, when the bridge is balanced, i.e., the galvanometer current is zero,

$$i\,\Delta R_\theta = i_\theta\,R_1$$

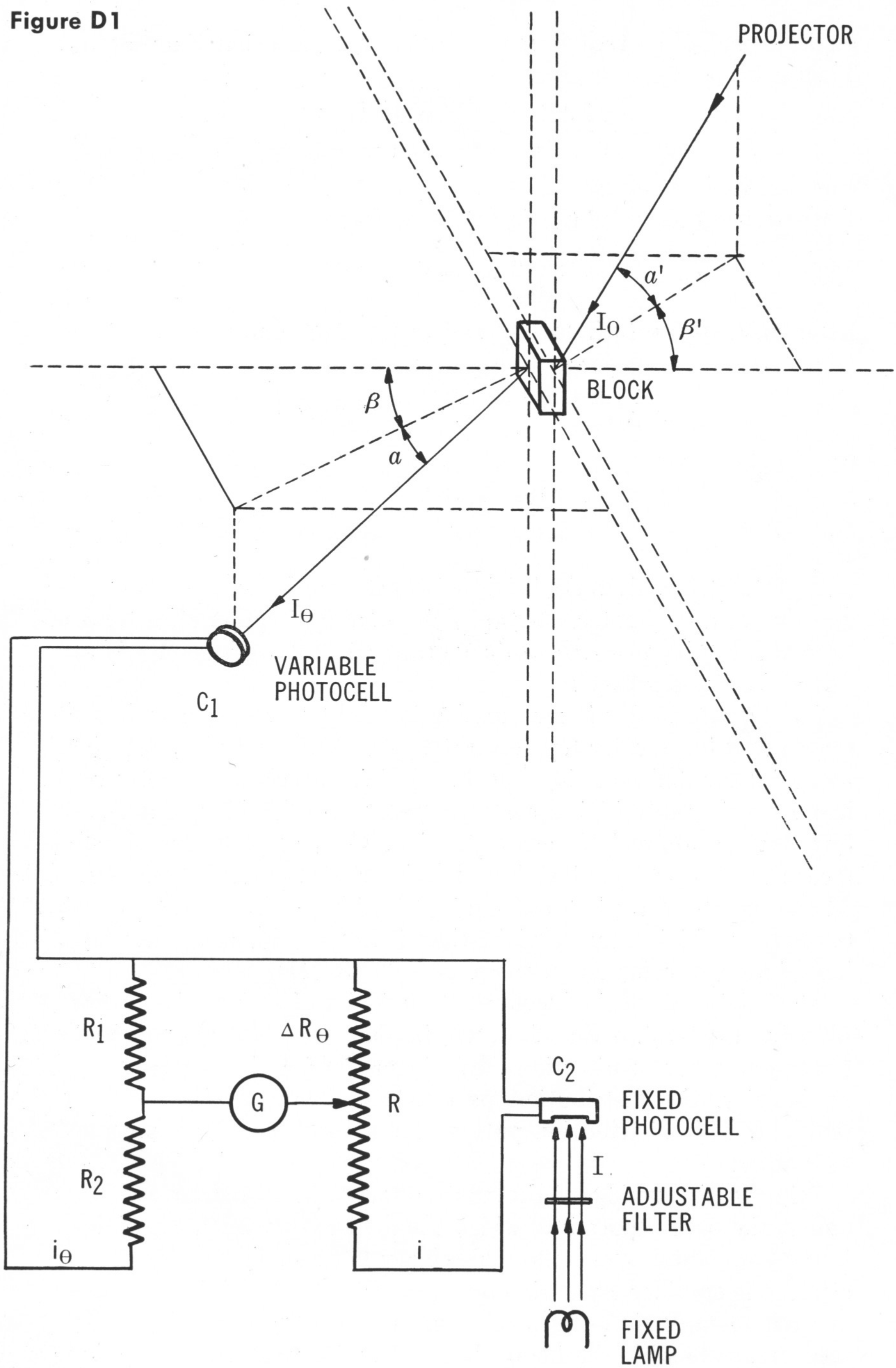
and

$$\frac{k\,I\,\Delta R_\theta}{r + R} = \frac{k'\,I_\theta\,R_1}{r' + R_1 + R_2}. \qquad \text{D1.2}$$

In a series of measurements involving the same block and the same exterior illumination, values of ΔR_θ are recorded for specified directions α and β. Occasional checks are made on the first reading of the series, and, if any change is noted, the reading is adjusted to its original value by changing some of R_2 to R_1, or vice versa.

After a series of measurements is completed, cell C_1 is equipped with a calibrated metallic film filter, moved to the position formerly occupied by the center of the exterior face of the block, and set in

Figure D1


PROJECTOR
a'
β'
I_0
BLOCK
β
a
I_Θ
VARIABLE PHOTOCELL
C_1
R_1
ΔR_Θ
G
R
C_2
FIXED PHOTOCELL
I
ADJUSTABLE FILTER
R_2
i_Θ
i
FIXED LAMP

angle α' and β' so that it measures the normal beam intensity I_o. Then a bridge balance gives

$$\frac{k\,I\,\Delta R_o}{r + R} = \frac{k'\,K\,I_o\,R_1}{r' + R_1 + R_2}, \qquad \text{D1.3}$$

where K is the filter factor.

By dividing Eq D1.2 by Eq D1.3, one has

$$\frac{\Delta R_\theta}{\Delta R_o} = \frac{I_\theta}{K\,I_o}. \qquad \text{D1.4}$$

After substituting I_θ from Eq D1.1 in Eq D1.4, one has

$$\frac{\Delta R_\theta}{\Delta R_o} K\,I_o = \frac{B_\theta\,A_b}{\pi R^2} \cos\alpha \cos\beta$$

or

$$B_\theta = \frac{\Delta R_\theta}{\Delta R_o}\;\frac{K\pi R^2}{A_b}\;\frac{I_o}{\cos\alpha\cos\beta}. \qquad \text{D1.5}$$

Eq D1.5, then, gives the brightness of the block face for a direction α, β due to a collimated beam of light from a direction α', β' with a normal intensity of I_o. If I_o is expressed in foot-candles B_θ is obtained in foot-lamberts.

The brightness B_θ is measured using light from an incandescent source that has a color temperature of about 2700° K; however, it is to be interpreted as being due to daylight which, on the average, has a considerably higher color temperature. This change does not introduce an appreciable error, provided the photocells used for the measurements are equipped with Viscor visual correction filters. The response of a photocell equipped with a Viscor filter is only 0.8 per cent higher for daylight than it is for light from a tungsten lamp having a color temperature of 2700° K (11).

The brightness of the block for a particular direction θ is obtained by using Eq D1.5, and such determinations can, of course, be made for any number of directions. But in predicting the daylighting of an interior, the distribution pattern for the transmitted light is required. This distribution pattern for a block can be determined as follows:

Prior to the taking of the measurements indicated by Eq D1.4 above, the entire space into which the light is transmitted is divided into a large number of solid angle elements ($\Delta\omega$). The center of each element is specified by a direction $\theta(\alpha, \beta)$, and the division is made in such manner that the total sum of the solid angles is 2π. The measurements with the hoop photocell are then made for all these directions, and the intensity measured is assigned to the small surface designated by the solid angle element $(\Delta\omega)_\theta$. If the imaginary hemisphere to which the solid angles refer is thought to be defined by the movement of the photocell hoop, then

$$(\Delta\omega)_\theta = \frac{\Delta a}{R^2}, \tag{D1.6}$$

where Δa is the small area to which I_θ is assigned. The luminous energy in each solid angle element is then given by

$$(\Delta F)_\theta = I_\theta \, \Delta a = I_\theta \, R^2 \, (\Delta\omega)_\theta, \tag{D1.7}$$

and the total light transmitted is given by

$$F = \sum (\Delta F)_\theta = \sum I_\theta \, R^2 \, (\Delta\omega)_\theta, \tag{D1.8}$$

when the summation is carried over the solid angle 2π.

If Eq D1.4 is used for I_θ, then Eq D1.8 becomes

$$F = \sum R^2 \frac{\Delta R_\theta}{\Delta R_\circ} K \, I_\circ \, (\Delta\omega)_\theta$$

$$= \frac{R^2 \, K \, I_\circ}{\Delta R_\circ} \sum \Delta R_\theta \, (\Delta\omega)_\theta \,. \tag{D1.9}$$

If Eq D1.1 is used, then

$$F = \sum \frac{B_\theta}{\pi} A_b \, (\Delta\omega)_\theta \cos \alpha \cos \beta$$

$$= \frac{A_b}{\pi} \sum B_\theta \, (\Delta\omega)_\theta \cos \alpha \cos \beta \tag{D1.10}$$

If one is not concerned with the distribution of the transmitted light but is interested only in the total amount of light transmitted, then such a series of measurements is unnecessary, since the integrating sphere can be used.

From Eq C1.8 (Appendix C), one observes that the photocell response of the integrating sphere is directly proportional to the total lumens admitted to the sphere interior. Thus, if the integrating sphere is set to receive all the light transmitted by the block from the artificial sun and the sphere photocell is connected into the bridge circuit in place of the hoop photocell C_1, the total transmission can be determined by a single reading as follows:

The total lumens incident on the exterior face of the block with the artificial sun in any position is

$$I_o \cos \theta' = I_o \cos \alpha' \cos \beta'$$

and the total lumens transmitted is

$$F(\theta') = A_b\, I_o\, T \cos \alpha' \cos \beta' \,, \qquad \text{D1.11}$$

where T is the fractional transmission for the direction θ'. When the bridge is balanced, one has

$$\frac{k\, I\, \Delta R(\theta')}{r + R} = \frac{k''\, R_1\, F(\theta')}{r' + R_1 + R_2} \,, \qquad \text{D1.12}$$

where again k, k″, r, and r′ are circuit constants. The block is removed and a normal intensity reading is taken as indicated by Eq D1.3. The artificial sun is moved to $\alpha' = \beta' = 0$, and again the normal intensity I_o' is determined. From these two readings the ratio of intensities is obtained, i.e.,

$$\frac{I_o}{I_o'} = p \,, \qquad \text{D1.13}$$

where p, in fact, is close to unity.

A diffusing plate, for which the total transmission of a collimated beam of light at normal incidence is known, is attached to the sphere opening and the sphere is set so that the plate is at the position at which I_o' was measured. The total lumens entering the sphere is

$$F_o = I_o'\, A_o\, T_o \,, \qquad \text{D1.14}$$

where A_o is the transmitting area of the diffusing plate and T_o its fractional transmission. Then a balanced bridge gives

$$\frac{k\, I\, \Delta R_o'}{r + R} = \frac{k''\, R_1\, F_o}{r' + R_1 + R_2} \,. \qquad \text{D1.15}$$

After dividing Eq D1.12 by D1.15, one has

$$\frac{\Delta R(\theta')}{\Delta R_o'} = \frac{F(\theta')}{F_o} \,,$$

which becomes with the aid of Eqs D1.11, D1.13, and D1.14

$$\frac{\Delta R(\theta')}{\Delta R_o'} = p\, \frac{A_b\, T \cos \alpha' \cos \beta'}{A_o T_o}$$

and

$$T = \frac{\Delta R(\theta')}{\Delta R_o'} \, \frac{A_o T_o}{p\, A_b \cos \alpha' \cos \beta'} \qquad \text{D1.16}$$

As a second method of determining total transmission, one can use a lamp, for which the total lumen output is known, in the integrating sphere and a corrected photocell for measuring the illumination of the exterior block face. Then one has two readings to which Eq

D1.12 applies, one with the sphere lamp off and the artificial sun effective and the other with the sphere lamp on and the artificial sun ineffective. This gives

$$\frac{\Delta R(\theta')}{\Delta R_k} = \frac{F(\theta')}{F_k} ,$$

where ΔR_k and F_k are the bridge reading and total lumens for the sphere lamp. Substituting for $F(\theta')$ from Eq D1.11 for which $I_o \cos \alpha' \cos \beta'$ has been measured in foot-candles with a corrected photocell, one has

$$\frac{\Delta R(\theta')}{\Delta R_k} = \frac{A_b\, T\, (I_o \cos \alpha' \cos \beta')}{F_k}$$

and

$$T = \frac{\Delta R(\theta')}{\Delta R_k} \frac{F_k A_b}{(I_o \cos \alpha' \cos \beta')} . \qquad \text{D1.17}$$

The Determination of Block Brightness and Transmission for Light from a Sky

The brightness and tranmission of a prismatic block must be known for the diffuse light from the sky as well as for the direct light from the sun if one is to do a complete and accurate job of predicting the performance of a particular installation. As previously mentioned, an artificial sky similar to the one involved could be constructed in the laboratory and the brightness and transmission for the block measured directly. Experience has indicated that this is a time-consuming job for one type of sky and actually a formidable task when a complete prediction procedure involves several types of skies. Therefore, a more direct method has been employed, and the accuracy of the results is sufficiently high for all practical purposes.

This procedure involves the measurement of brightness and transmission, in accordance with the methods described above, for collimated beams of light from a sufficiently large number of directions for a complete coverage of a sky when a definite portion of the sky is assigned to each direction selected. Assume that a vertical surface receives light from π steradians of sky and that this sky vault is divided into 81 segments, each segment being 10 altitude degrees high and 10 azimuth degrees wide. The centers of the segments can be represented by the altitude angles α_i' and the azimuth angles β_i', where α_i' varies by 10-degree intervals from 5 degrees to 85 degrees and β_i' varies by 10-degree intervals from -85 degrees to $+85$ degrees.

The vertical surface illumination contributed by each sky segment can be calculated by adapting Eq A1.1, i. e., $\cos\theta = 1$, $\cos\theta' = \cos\alpha'\cos\beta'$, $d\sigma = r^2\cos\alpha'\,d\alpha'\,d\beta'$, and $B = k_o B_o$. Thus

$$\Delta E_v = \frac{k_o\,B_o}{\pi}\int_S \cos^2\alpha'\cos\beta'\,d\alpha'\,d\beta'\,, \qquad \text{D2.1}$$

where the integration extends over the entire segment and k_o is the ratio of the brightness of the segment to the zenith brightness B_o. This integration gives

$$(\Delta E_v)_i = \frac{k_o\,B_o}{\pi}\sin\beta'\Big]_{\beta_i'-5^\circ}^{\beta_i+5^\circ}\left(\frac{\alpha'}{2}+\frac{\sin 2\alpha'}{4}\right)\Big]_{\alpha_i'-5^\circ}^{\alpha_i'+5^\circ} \qquad \text{D2.2}$$

Now, from Eq D1.5 one has that

$$(B\theta)_i = \left(\frac{\Delta R\theta}{\Delta R_o}\right)_i \frac{K\pi R^2}{A_b}\,\frac{(I_o)_i}{\cos\alpha\cos\beta}\,, \qquad \text{D2.3}$$

where $(B\theta)_i$ is the brightness of the block from a direction θ, specified by the observer altitude α and the observer azimuth β for a collimated beam of light from a source direction specified by α_i' and β_i', $(I_o)_i$ is the normal beam intensity for the direction α_i' and β_i' and $(\Delta R\theta/\Delta R_o)_i$ is the ratio of bridge readings for this situation. If this block brightness is to be considered as due to the segment of sky with center at α_i', and β_i', then $(I_o)_i\cos\alpha_i'\cos\beta_i'$ must be equal to $(\Delta E_v)_i$ as given by Eq D2.2. Thus, Eq D2.3 becomes

$$(B\theta)_i = \left(\frac{\Delta R\theta}{\Delta R_o}\right)_i \frac{K\pi R^2}{A_b}\,\frac{(\Delta E_v)_i}{\cos\alpha\cos\beta\cos\alpha_i'\cos\beta_i'} \qquad \text{D2.4}$$

and the total brightness of the block in the direction θ is

$$\sum (B\theta)_i = \frac{K\pi R^2}{A_b\cos\alpha\cos\beta}\sum_i\left(\frac{\Delta R\theta}{\Delta R_o}\right)\frac{(\Delta E_v)_i}{\cos\alpha_i'\cos\beta_i'} \qquad \text{D2.5}$$

where the measurement of $\Delta R\theta/\Delta R_o$ and the summation includes the entire sky under consideration. The quantity $\Sigma\,(\Delta E_v)_i$ is, of course, the exterior vertical surface illumination due to the direct light from the sky.

For example, consider the evaluation of some of these quantities for the case of an overcast sky. According to published meteorological data the brightness pattern of an overcast sky is given by Fig. B5. When this sky is considered to consist of 81 segments, each 10 altitude degrees high and 10 azimuth degrees wide, and the brightness of the center of each segment, relative to the zenith, is taken from Fig. B5, one has the chart shown in Table D1 for a quarter sky. Consider that the total sky effective in lighting the exterior of the

TABLE D1

OVERCAST SKY—RELATIVE BRIGHTNESS (Zenith = 1.)

$$k_\circ = \frac{B}{B_\circ}$$

Azimuth Angle—β'

Altitude Angle—α'	5	15	25	35	45	55	65	75	85
5	0.42	0.41	0.40	0.39	0.38	0.38	0.37	0.36	0.36
15	0.54	0.55	0.54	0.53	0.51	0.50	0.51	0.52	0.51
25	0.65	0.65	0.66	0.67	0.67	0.66	0.65	0.65	0.64
35	0.77	0.77	0.77	0.77	0.76	0.76	0.74	0.74	0.73
45	0.91	0.87	0.87	0.84	0.84	0.83	0.82	0.82	0.82
55	0.94	0.94	0.93	0.92	0.91	0.90	0.88	0.88	0.87
65	0.98	0.98	0.97	0.96	0.95	0.94	0.93	0.93	0.92
75	1.00	1.00	1.00	1.00	0.99	0.98	0.95	0.95	0.95
85	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.98

TABLE D2

VALUES OF $\left.\frac{\sin \beta'}{\pi}\right]_{\beta_i'-5°}^{\beta_i'+5°} \left.\left(\frac{\alpha'}{2} + \frac{\sin 2\alpha'}{4}\right)\right]_{\alpha_i'-5°}^{\alpha_i'+5°}$

Azimuth Angle—β'

Altitude Angle—α'	5	15	25	35	45	55	65	75	85	Totals
5	0.00953	0.00925	0.00868	0.00784	0.00677	0.00549	0.00405	0.00247	0.00083	0.05491
15	0.00897	0.00870	0.00816	0.00738	0.00637	0.00516	0.00381	0.00233	0.00078	0.05166
25	0.00791	0.00766	0.00719	0.00649	0.00561	0.00455	0.00335	0.00205	0.00069	0.04550
35	0.00646	0.00626	0.00587	0.00531	0.00458	0.00372	0.00274	0.00167	0.00056	0.03717
45	0.00482	0.00467	0.00438	0.00396	0.00342	0.00277	0.00204	0.00125	0.00042	0.02773
55	0.00317	0.00308	0.00289	0.00261	0.00225	0.00183	0.00134	0.00082	0.00027	0.01826
65	0.00173	0.00168	0.00158	0.00142	0.00123	0.00099	0.00073	0.00044	0.00014	0.00994
75	0.00066	0.00064	0.00060	0.00054	0.00047	0.00038	0.00028	0.00017	0.00006	0.00380
85	0.00010	0.00009	0.00009	0.00008	0.00007	0.00005	0.00004	0.00003	0.00001	0.00056
Totals	0.04335	0.04203	0.03944	0.03563	0.03077	0.02494	0.01838	0.01123	0.00376	0.25000

TABLE D3

VALUES OF $\dfrac{(\Delta E_v)_i}{B_o}$

Azimuth Angle—β'

Altitude Angle—α'	5	15	25	35	45	55	65	75	85	Totals
5	0.0040	0.0038	0.0035	0.0030	0.0026	0.0021	0.0015	0.0009	0.0003	0.0217
15	0.0048	0.0048	0.0044	0.0039	0.0032	0.0026	0.0019	0.0012	0.0004	0.0272
25	0.0051	0.0050	0.0047	0.0043	0.0038	0.0030	0.0022	0.0013	0.0005	0.0299
35	0.0050	0.0048	0.0045	0.0041	0.0035	0.0028	0.0020	0.0012	0.0004	0.0283
45	0.0044	0.0041	0.0038	0.0033	0.0029	0.0023	0.0017	0.0010	0.0003	0.0238
55	0.0030	0.0029	0.0027	0.0024	0.0020	0.0017	0.0012	0.0007	0.0002	0.0168
65	0.0017	0.0016	0.0015	0.0014	0.0012	0.0009	0.0007	0.0004	0.0001	0.0091
75	0.0007	0.0006	0.0006	0.0005	0.0005	0.0004	0.0003	0.0002	0.0001	0.0039
85	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0010
Totals	0.0288	0.0277	0.0258	0.0230	0.0198	0.0159	0.0115	0.0069	0.0023	0.1617

fenestration is represented by two of these charts. Table D1, then, gives the values of k_o of Eq D2.2 and Table D2 shows the values of

$$\frac{\sin \beta'}{\pi}\Bigg]_{\beta_i'-5^\circ}^{\beta_i'+5^\circ} \left(\frac{\alpha'}{2} + \frac{\sin 2\alpha'}{4}\right)\Bigg]_{\alpha_i'-5^\circ}^{\alpha_i'+5^\circ}$$

for the same sky divisions. If the terms of Table D1 are multiplied by the corresponding terms of Table D2, one obtains values of $(\Delta E_v)_i/B_o$ for Eq D2.2 as listed in Table D3.

The total vertical surface illumination, owing to light from an unobstructed sky having a uniform brightness of B foot-lamberts, is B/2 foot-candles. This result is obtained by Eq D1.17, when the integration extends from $\alpha' = 0$ to $\alpha' = \pi/2$ and from $\beta' = -\pi/2$ to $\beta' = +\pi/2$ and $k_o\ B_o = B$. Also twice the sum of the terms of Table D2 is ½, which is in agreement.

The total vertical surface illumination due to light from an unobstructed overcast sky having a zenith brightness of B_o foot-lamberts and a brightness pattern as indicated by Table D1 is twice the sum of the terms of Table D3 or 0.32 B_o foot-candles.

In order to determine the brightness of a block in a specific direction due to this overcast sky, measurements are taken and a table drawn up for $(\Delta R_\theta/\Delta R_o)_i$. Then, values of $(B_\theta)_i$ are calculated using Eq D2.5, the values of $(\Delta E_v)_i$ from Table D3, and the measured values of $(\Delta R_\theta/\Delta R_o)_i$. The sum of these values (S) for the entire sky gives the block brightness in terms of the zenith brightness or $B_\theta = \Sigma (B_\theta)_i = SB_o$. If B_θ is desired for an overcast sky that produces a vertical surface illumination of 1000 foot-candles, then, since $1000 = 0.32\ B_o$, $B_o = 3100$ foot-lamberts and $B_\theta = 3100\ S$ foot-lamberts.

The total amount of light transmitted by a prismatic glass block from a particular sky can be determined in a similar manner. The fractional transmission of a block for a collimated beam of light from each direction, α_i' and β_i', can be determined by integrating sphere measurements and calculation by Eq D1.15 or Eq D1.16. Thus one obtains a table of values of T_i. From Eq D2.2 one has a table of values for $(\Delta E_v)_i$. Then, the total amount of light incident on the exterior of the block from the entire sky is $\Sigma\ (\Delta E_v)_i$, the total amount transmitted is $\Sigma\ T_1\ (\Delta E_v)_i$, and the fractional transmission is

$$T_s = \frac{\sum T_i(\Delta E_v)_i}{\sum (\Delta E_v)_i}. \qquad \text{D2.}$$

The Prediction of Daylighting

The prediction of the daylighting of building interiors can be based either on the direct light from the fenestration alone or on the effect of multiple reflections within the room as well. If only the direct light is considered, then the calculated illumination for a position remote from the fenestration in a lightly decorated room may be only a small fraction of the correct value. Because of this, the prediction work involving prismatic glass block has always included the effect of the scattered light.

The brightness and transmission characteristics of prismatic glass block have been determined in great detail for a variety of sun and sky conditions. These data have included the amount of light transmitted into particular solid angles, the determination being in accordance with Eq D1.7. Through these light-distribution patterns the approximate amount of light initially incident on all room surfaces, when the entire block panel is taken into account, can be determined. If these room surfaces are divided into narrow panels parallel to the fenestration, then the illumination at a particular station due to direct light from the fenestration and first reflections from the panels can be calculated through the theory of surface sources. Experimental work with a quarter-scale model indicated that for a room fenestrated with functional glass block the increase of illumination due to reflections of light beyond the first reflection was practically the same for all parts of the room.

Since the total amount of light (F) entering the room is known and since all this light is lost by absorption or transmission to the exterior, then

$$F = \sum I_i \alpha_i ,$$

where I is the final illumination of the area that has an absorption α and the summation includes all the important room surfaces. The value of I for the task is greater than the illumination due to direct light from the fenestration and first reflections, and consequently the difference is the result of the remainder of the reflections.

The presentation of more details of this prediction method would involve the inclusion of numerous pages of block data. In the interest of space, then, only this brief outline will be included.

Bibliography

1. Recommended Practice of Daylighting, Illuminating Engineering Society, 1950.

2. "Variations in the Total and Luminous Solar Radiation with Geographical Position in the United States," Herbert H. Kimball, U. S. Weather Review, November, 1919.

3. "Daylight Illumination on Horizontal, Vertical and Sloping Surfaces," Herbert H. Kimball and Irving F. Hand, U. S. Weather Review, December, 1922.

4. "Variations of Intensities of the Visible and of the Ultraviolet in Sunlight and in Skylight," William Kunerth and Russell D. Miller, *Transactions,* I. E. S. *27* (1932), 82-94.

5. "Visible and Ultraviolet in the Light Obtained from the Sun," William Kunerth and Russell D. Miller, *Transactions,* I. E. S. *28* (1933), 347-353.

6. "A Time Analysis of Sunshine," Frank Benford and John E. Boch, *Transactions,* I. E. S. *34* (1939), 200-218.

7. "The Calculation of Illumination from Sun and Sky," E. Elvegard and G. Sjostedt, *Transactions,* I. E. S. *35* (1940), 333-342.

8. "Duration and Intensity of Sunshine," Frank Benford, *Transactions,* I. E. S. *42* (1947), 527-544; *Transactions,* I. E. S. *42* (1947), 877-889; *Transactions,* I. E. S. *43* (1948), 533-546.

9. "Mitteilungen der Sternwarte Heidelberg," Nr. 4 (1904), A. Bemporad, *Meteorol. Zeitschr.* 24 (1907), 309.

10. Scientific Papers of the Bureau of Standards, No. 415.

11. Technical Data on Weston Photronic Cells, Weston Electrical Instrument Corporation, Cir. B20B, October, 1944.

12. "Brightness," J. R. Cravath, *Transactions,* I. E. S. 9 (1914), 394-412.

13. "Sky Brightness and Daylight Illumination Measurements," Report of the Committee on Sky Brightness, H. H. Kimball, Chairman, *Transactions,* I. E. S. *16* (1921), 255-283.

14. "Daylight Illumination on Horizontal, Vertical and Sloping Surfaces," Report of the Committee on Sky Brightness, H. H. Kimball, Chairman, *Transactions,* I. E. S. *18* (1923), 434-474.

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